Non-Newtonian hidden-variable models in quantum mechanics

Kamil Nalikowski

Supervisor prof. Marek Czachor

$$P_{0_1 0_2} = \frac{1}{2}\sin^2\frac{\alpha-\beta}{2} = \int_0^{(2\pi)'} \chi_{0_1}(x) \odot^1 \chi_{0_2}(x) \odot^1 \rho(x) \mathrm{D}_1 x$$

Overview

- Brief introduction
- Bell Clauser Horne integral
- Quick course of non-Newtonian mathematics

doi:10.1038/nature13132

The ultimate physical limits of privacy

Artur Ekert[1,2] & Renato Renner[3]

Among those who make a living from the science of secrecy, worry and paranoia are just signs of professionalism. Can we protect our secrets against those who wield superior technological powers? Can we trust those who provide us with tools for protection? Can we even trust ourselves, our own freedom of choice? Recent developments in quantum cryptography show that some of these questions can be addressed and discussed in precise and operational terms, suggesting that privacy is indeed possible under surprisingly weak assumptions.

PERSPECTIVE

doi:10.1038/nature13132

The ultimate physical limits of privacy

Artur Ekert[1,2] & Renato Renner[3]

Among those who make a living from the science of secrecy, worry and paranoia are just signs of professionalism. Can we protect our secrets against those who wield superior technological powers? Can we trust those who provide us with tools for protection? Can we even trust ourselves, our own freedom of choice? Recent developments in quantum cryptography show that some of these questions can be addressed and discussed in precise and operational terms, suggesting that privacy is indeed possible under surprisingly weak assumptions.

Bell Clauser Horne integral

$$P_{A_1 A_2} = \int \chi_{A_1}(x)\chi_{A_2}(x)\rho(x)\mathrm{d}x$$

Bell Clauser Horne integral

$$P_{A_1 A_2} = \int \chi_{A_1}(x) \cdot \chi_{A_2}(x) \cdot \rho(x) \mathrm{d}x$$

Bell Clauser Horne integral

"among those who make a living from the science of secrecy, worry and paranoia are just signs of professionalism” – Artur Ekert

$$P_{A_1 A_2} = \int \chi_{A_1}(x) \odot^l \chi_{A_2}(x) \odot^l \rho(x) \mathrm{D}_l x$$

$$\frac{2}{3}c + \frac{2}{3}c = \frac{4}{3}c \qquad \frac{2}{3}c \oplus \frac{2}{3}c = \frac{12}{13}c$$

Bell Clauser Horne integral

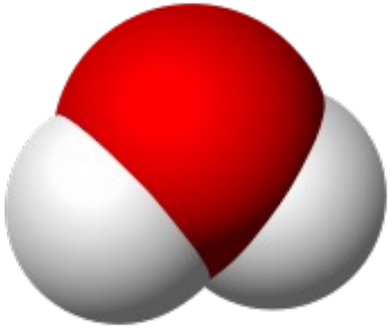
"among those who make a living from the science of secrecy, worry and paranoia are just signs of professionalism" – Artur Ekert

$$P_{A_1 A_2} = \int \chi_{A_1}(x) \odot^l \chi_{A_2}(x) \odot^l \rho(x) \mathrm{D}_l x$$

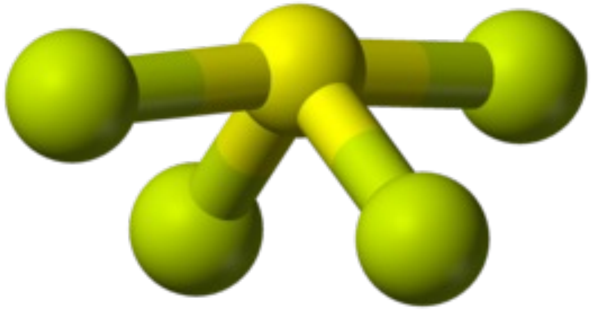
$$\frac{2}{3}c + \frac{2}{3}c = \frac{4}{3}c \qquad \frac{2}{3}c \oplus \frac{2}{3}c = \frac{12}{13}c$$

Bell Clauser Horne integral

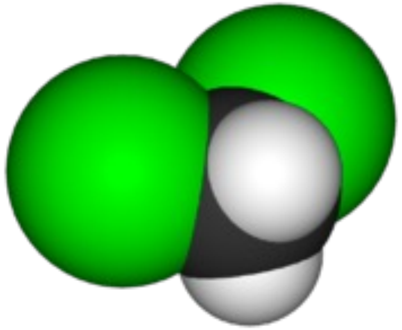
- They correspond to the same symmetry group
- There exist isomorphisms beetwen their representations
- Still, they are physically different



H_2O



SF_4



CH_2Cl_2

Quick course of non-Newtonian maths

$f_{\mathbb{X}} : \mathbb{X} \to \mathbb{R}$

$$x \oplus_{\mathbb{X}} y = f_{\mathbb{X}}^{-1}\left(f_{\mathbb{X}}(x) + f_{\mathbb{X}}(y)\right),$$

$$x \ominus_{\mathbb{X}} y = f_{\mathbb{X}}^{-1}\left(f_{\mathbb{X}}(x) - f_{\mathbb{X}}(y)\right),$$

$$x \odot_{\mathbb{X}} y = f_{\mathbb{X}}^{-1}\left(f_{\mathbb{X}}(x) \cdot f_{\mathbb{X}}(y)\right),$$

$$x \oslash_{\mathbb{X}} y = f_{\mathbb{X}}^{-1}\left(f_{\mathbb{X}}(x) / f_{\mathbb{X}}(y)\right).$$

Quick course of non-Newtonian maths

$f : \mathbb{R} \to \mathbb{R}. \longrightarrow f^k : \mathbb{R} \to \mathbb{R}.$

$$x \oplus^k y = f^{-k}\left(f^k(x) + f^k(y)\right),$$
$$x \ominus^k y = f^{-k}\left(f^k(x) - f^k(y)\right),$$
$$x \odot^k y = f^{-k}\left(f^k(x) \cdot f^k(y)\right),$$
$$x \oslash^k y = f^{-k}\left(f^k(x) / f^k(y)\right).$$

Quick course of non-Newtonian maths

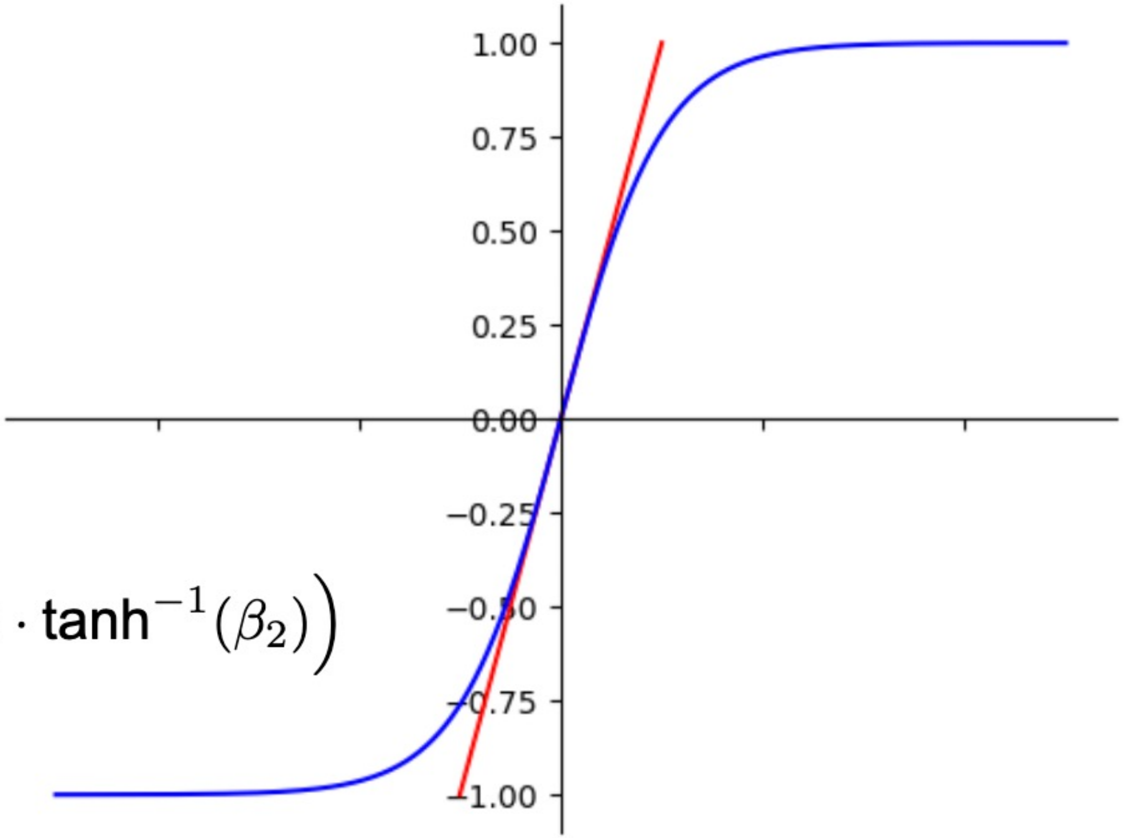
- Relativistic law of addition

$$\beta_1 \oplus \beta_2 = \tanh\left(\tanh^{-1}(\beta_1) + \tanh^{-1}(\beta_2)\right)$$

$$f_{\mathbb{X}} : [-1, 1] \to \mathbb{R}$$

$$f_{\mathbb{X}} = \tanh^{-1}(\beta)$$

$$\beta_1 \odot \beta_2 = \tanh\left(\tanh^{-1}(\beta_1) \cdot \tanh^{-1}(\beta_2)\right)$$



Quick course of non-Newtonian maths

$$\frac{\mathsf{D}_l A(x)}{\mathsf{D}_k x} = g^l\left(\frac{\mathsf{d}\tilde{A}\big(f^k(x)\big)}{\mathsf{d}f^k(x)}\right) = g^l\left(\frac{\mathsf{d}\tilde{A}(r)}{\mathsf{d}r}\right)$$

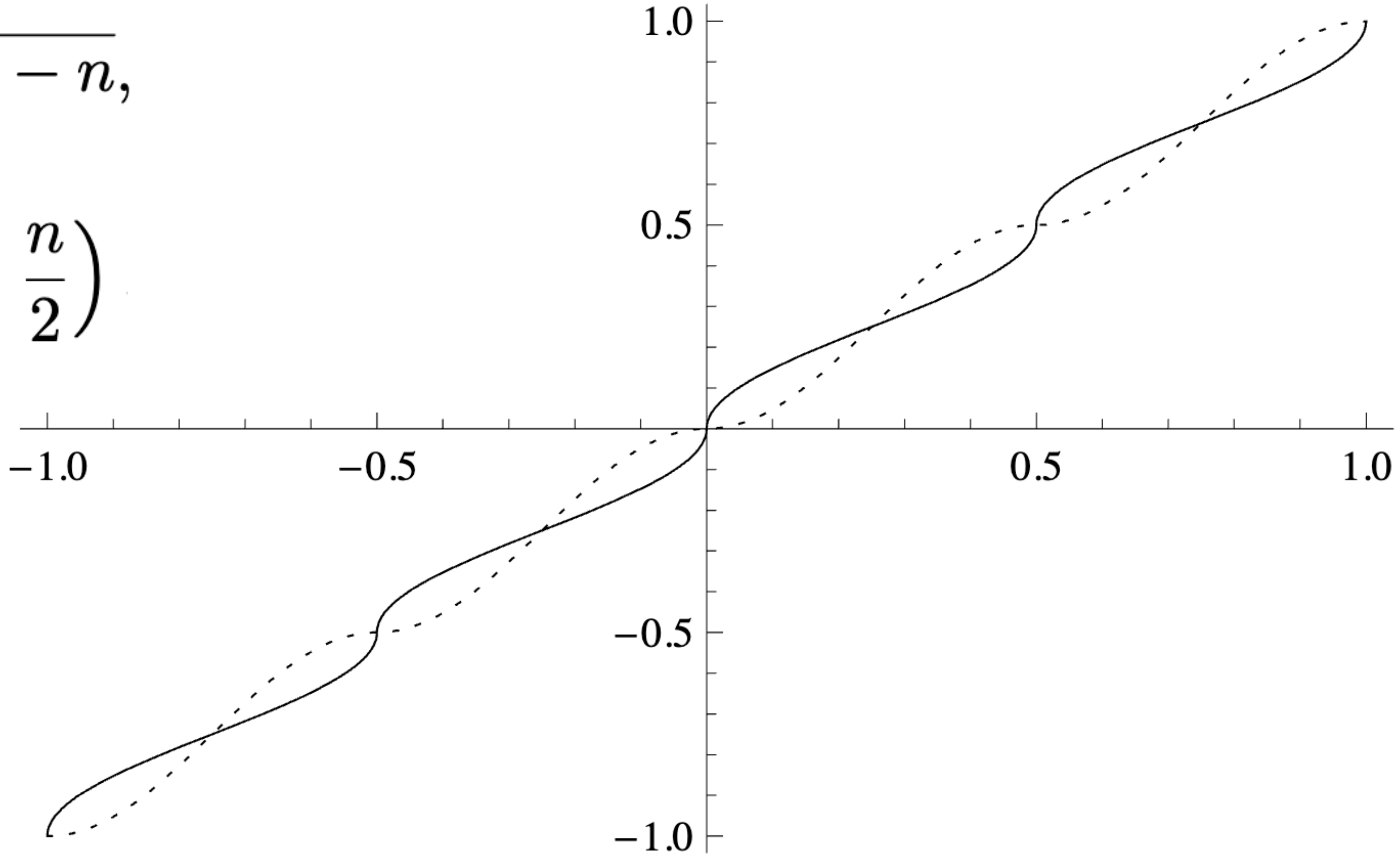
$$\int_a^b A(x)\mathsf{D}_k x = g^l\left(\int_{f^k(a)}^{f^k(b)} \tilde{A}(r)\mathsf{d}r\right)$$

Singlet-State Probabilities arithmetic

$$f(x) = \frac{n}{2} + \frac{1}{\pi} \arcsin \sqrt{2x - n},$$

$$g(x) = \frac{n}{2} + \frac{1}{2} \sin^2 \pi \left(x - \frac{n}{2} \right)$$

$$\frac{n}{2} \le x < \frac{n+1}{2},\, n \in \mathbb{Z}.$$



Singlet-State Probabilities arithmetic

$$P_{0_1 0_2} = \frac{1}{2}\sin^2\frac{\alpha-\beta}{2} = \int_0^{(2\pi)'} \chi_{0_1}(x) \odot^1 \chi_{0_2}(x) \odot^1 \rho(x) \mathrm{D}_1 x$$

$$P_{0_1 1_2} = \frac{1}{2}\cos^2\frac{\alpha-\beta}{2} = \int_0^{(2\pi)'} \chi_{0_1}(x) \odot^1 \chi_{1_2}(x) \odot^1 \rho(x) \mathrm{D}_1 x$$

$$P_{1_1 0_2} = \frac{1}{2}\cos^2\frac{\alpha-\beta}{2} = \int_0^{(2\pi)'} \chi_{1_1}(x) \odot^1 \chi_{0_2}(x) \odot^1 \rho(x) \mathrm{D}_1 x$$

$$P_{1_1 1_2} = \frac{1}{2}\sin^2\frac{\alpha-\beta}{2} = \int_0^{(2\pi)'} \chi_{1_1}(x) \odot^1 \chi_{1_2}(x) \odot^1 \rho(x) \mathrm{D}_1 x$$

Quantum Physics

[Submitted on 25 May 2021 (v1), last revised 13 Jul 2022 (this version, v4)]

Imitating quantum probabilities: Beyond Bell's theorem and Tsirelson bounds

Marek Czachor, Kamil Nalikowski

Local hidden-variable model of singlet-state correlations discussed in M. Czachor, Arithmetic loophole in Bell's Theorem: Overlooked threat to entangled-state quantum cryptography, Acta Phys. Polon. A 139, 70-83 (2021), is shown to be a particular case of an infinite hierarchy of local hidden-variable models based on an infinite hierarchy of calculi. Violation of Bell-type inequalities is shown to be a `confusion of languages' problem, a result of mixing different but neighboring levels of the hierarchy. Mixing of non-neighboring levels results in violations beyond the Tsirelson bounds.

Comments: Version published online in Foundations of Science. A talk related to this paper can be watched at this https URL

Subjects: **Quantum Physics (quant-ph)**

Cite as: arXiv:2105.12728 **[quant-ph]**

(or arXiv:2105.12728v4 **[quant-ph]** for this version)

https://doi.org/10.48550/arXiv.2105.12728

Related DOI: https://doi.org/10.1007/s10699-022-09856-y

Foundations of Science

https://doi.org/10.1007/s10699-022-09856-y

Imitating Quantum Probabilities: Beyond Bell's Theorem and Tsirelson Bounds

Marek Czachor[1] · Kamil Nalikowski[1]

Accepted: 15 June 2022