Mitigation of coherent measurement errors based on characterisation of quantum devices

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MEASUREMENT

$\rho : \mathcal{H} \to \mathcal{H}$, $\mathcal{H} \simeq \mathbb{C}^d$, $d < \infty$

$$\mathbf{M} = \{M^{(k)}\}_{k=1}^{n}, \quad \forall_k M^{(k)} \geq 0, \quad \sum_{k=1}^{n} M^{(k)} = \mathbb{1}_d, \qquad \left(\forall_{k,l} M^{(k)} M^{(l)} = \delta_{kl} M^{(k)}\right)$$

PROJECTIVE

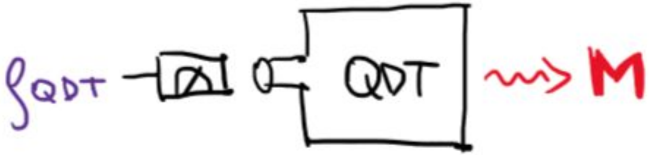
BORN'S RULE:

$$p(k \mid \rho, \mathbf{M}) = \mathrm{Tr}(\rho \cdot M^{(k)})$$

computational basis: $\{|\mathbf{x}\rangle\}_{\mathbf{x} \in \{0,1\}^n} = \{|i\rangle\}_{i=1}^{2^n}$ eg: $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$

computational basis POVM: $\{|i\rangle\langle i|\}_{i=1}^{2^n}$ eg: $\mathbf{P} = \{P^{(0)}, P^{(1)}\} = \left\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right\}$

QUANTUM DETECTOR TOMOGRAPHY



e.g.: ideal: $\mathbf{P} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$

noisy: $\mathbf{M} = \left\{ \begin{bmatrix} 1-r & 0 \\ 0 & q \end{bmatrix}, \begin{bmatrix} r & 0 \\ 0 & 1-q \end{bmatrix} \right\}$

CLASSICAL ERROR

$$\Lambda = \begin{bmatrix} 1-r & q \\ r & 1-q \end{bmatrix}, \quad \Lambda_{kl} = p(k|l) \qquad M^{(k)} = \sum_l p(k|l) P^{(k)}$$

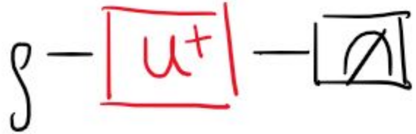
$$\mathbf{p}^{noisy} = \left[\sum_l \Lambda_{1l} \mathrm{Tr}\left(M^{(l)} \rho\right), \dots \right]^T = \Lambda \mathbf{p}^{ideal} \quad \Rightarrow \quad \mathbf{p}^{ideal} = \Lambda^{-1} \mathbf{p}^{noisy}$$

COHERENT ERROR

$$\mathbf{M} = \left\{ \begin{bmatrix} 1-r & \varepsilon \\ \bar{\varepsilon} & q \end{bmatrix}, \begin{bmatrix} r & -\varepsilon \\ -\bar{\varepsilon} & 1-q \end{bmatrix} \right\}$$

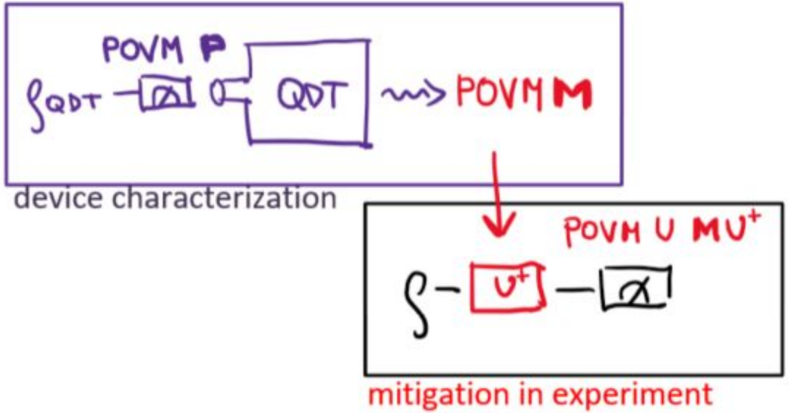
$$\mathbf{p}^{ideal} = \Lambda^{-1} \mathbf{p}^{noisy} + \Delta$$

UNITARY BEFORE MEASUREMENT



$$\rho \to U^\dagger \rho U$$

$$p(k \mid U^\dagger \rho U, \mathbf{M}) = \mathrm{Tr}\left(U^\dagger \rho U \, M^{(k)}\right) = \mathrm{Tr}\left(\rho U \, M^{(k)} U^\dagger\right) = p(k \mid \rho, U \mathbf{M} U^\dagger)$$



OPTIMIZATION PROBLEM

$$\underset{U}{\text{optimize}} \quad f(U\mathbf{M}U^{+})$$

$$\text{s.t.} \quad U \in \mathbf{S} \subset \mathbf{SU}(2^{n})$$

DISTANCES

$$\| M \|_{HS} = \sqrt{Tr(M^{+} M)} = \sqrt{\sum_{i,j} |m_{ij}|^2}$$

$$D_{TV}(\mathbf{p}, \mathbf{q}) = \frac{1}{2} \sum_{k=1}^{m} |p_k - q_k|$$

$$D_{op}(\mathbf{M}, \mathbf{N}) = \max_{\rho} D_{TV}(\mathbf{p_M}(\rho), \mathbf{p_N}(\rho))$$

$$D_{AVG}(\mathbf{M}, \mathbf{N}) = \frac{1}{2d} \sum_{k=1}^{m} \sqrt{Tr[(M^{(k)} - N^{(k)})^2] + Tr^2[M^{(k)} - N^{(k)}]}$$

$$D_{TV}(\Lambda^{-1}\mathbf{p}^{exp}, \mathbf{p}^{ideal}) \leq \| \Lambda^{-1} \|_{1 \to 1} D_{op}(\mathbf{M}, \Lambda \mathbf{P}) =: upb_{TV}(\mathbf{M})$$

COST FUNCTIONS

interpretation	worst–case	average–case	heuristic
measure of coherent error	$D^{dep}_{op}(UMU^\dagger)$ (2.3)	$D^{dep}_{avg}(UMU^\dagger)$ (2.4)	$\mathrm{Off}^{diag}(UMU^\dagger)$ (2.5)
distance from computational basis POVM	$D^{comp}_{op}(UMU^\dagger)$ (2.8)	$D^{comp}_{avg}(UMU^\dagger)$ (2.9)	$\mathrm{Ovl}^{comp}(UMU^\dagger)$ (2.10)
upper–bound on error in classical noise mitigation	$\mathrm{upb}_{TV}(UMU^\dagger)$ (2.6)	$\mathrm{upb}_{avg}(UMU^\dagger)$ (2.7)	

$$\sum_{k=1}^{m} \| U M^{(k)} U^{+} - \mathrm{diag}(U M^{(k)} U^{+}) \|_{HS}^{2}$$

$$\sum_{k=1}^{m} \langle k | M^{(k)} | k \rangle$$

$$\frac{1}{2\sqrt{m(m+1)}} \sum_{j=1}^{m} \sqrt{\mathrm{Tr}(\Theta^{(j)2}) + \mathrm{Tr}^{2}(\Theta^{(j)})}$$

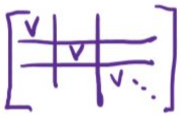
$$\Theta^{(j)} = \sum_{k=j}^{m} [\Lambda^{-1}_{jk} [UMU^{+} - \mathrm{diag}(UMU^{+})]$$

ANALYTICAL OPTIMIZATION

MINIMIZE:

$$\mathrm{Off}(UMU^\dagger) = \sum_{k=1}^{m} \| UM^{(k)}U^\dagger - \mathrm{diag}(UM^{(k)}U^\dagger)\|_{\mathrm{HS}}^2$$

$$U = \mathbb{1}_{2^{n-1}} \otimes V =$$



J.-F. Cardoso and A. Souloumiac, "Jacobi angles for simultaneous diagonalization", SIAM Journal on Matrix Analysis and Applications **17**, 161–164 (1996).

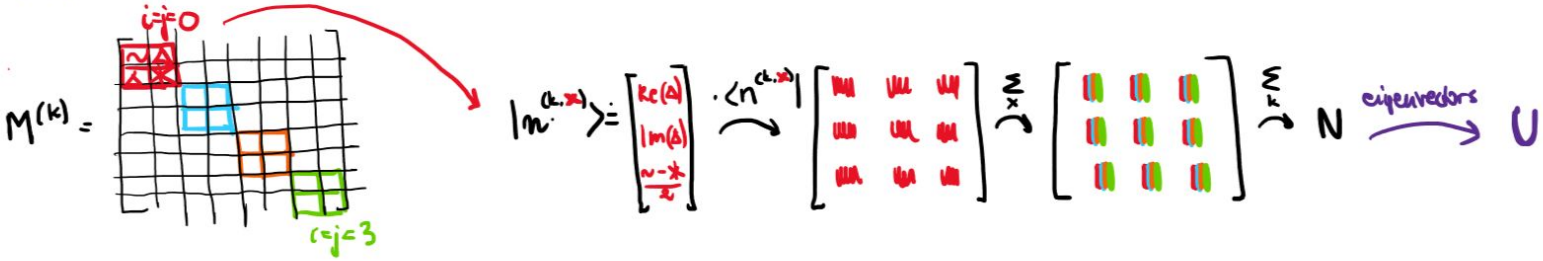
EIGENPROBLEM FOR 3X3 MATRIX:

$$N(\mathrm{M}) := \sum_{k=1}^{m} \sum_{\mathbf{x}\in\{0,1\}^{n-1}} |\mathbf{n}^{(k,\mathbf{x})}\rangle\langle\mathbf{n}^{(k,\mathbf{x})}|$$

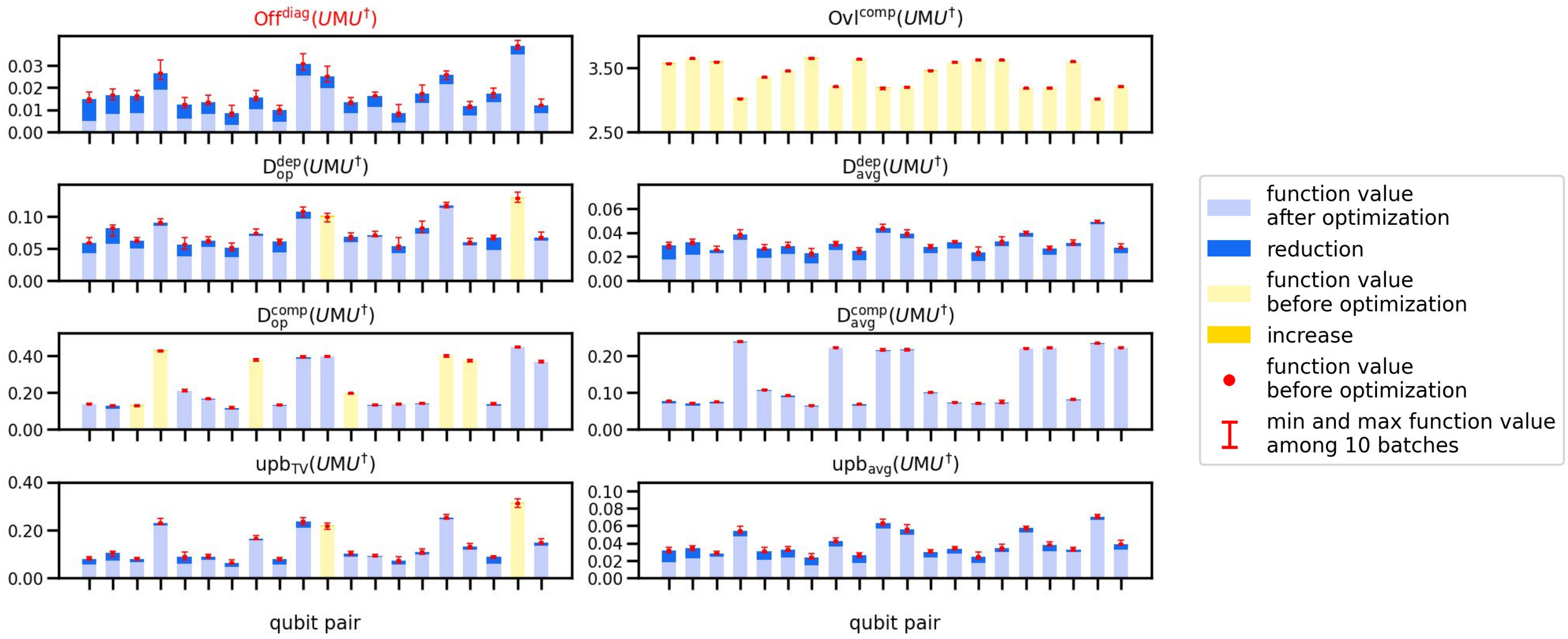
$$|\mathbf{n}^{(k,\mathbf{x})}\rangle := [\mathrm{Re}(m_{21}^{(k,\mathbf{x},\mathbf{x})}), \mathrm{Im}(m_{21}^{(k,\mathbf{x},\mathbf{x})}), \frac{1}{2}(m_{11}^{(k,\mathbf{x},\mathbf{x})} - m_{22}^{(k,\mathbf{x},\mathbf{x})})]^{\mathrm{T}} \in \mathbb{R}^3$$

$$M^{(k)} = \sum_{\substack{\mathbf{x},\mathbf{y}\in\{0,1\}^{n-1}\\ i,j\in\{0,1\}}} m_{ij}^{(k,\mathbf{x},\mathbf{y})} |\mathbf{x}\rangle\langle\mathbf{y}| \otimes |i\rangle\langle j|$$

e.g. for 3 qubits:



Rigetti's 80-qubit ASPEN-M-1 device



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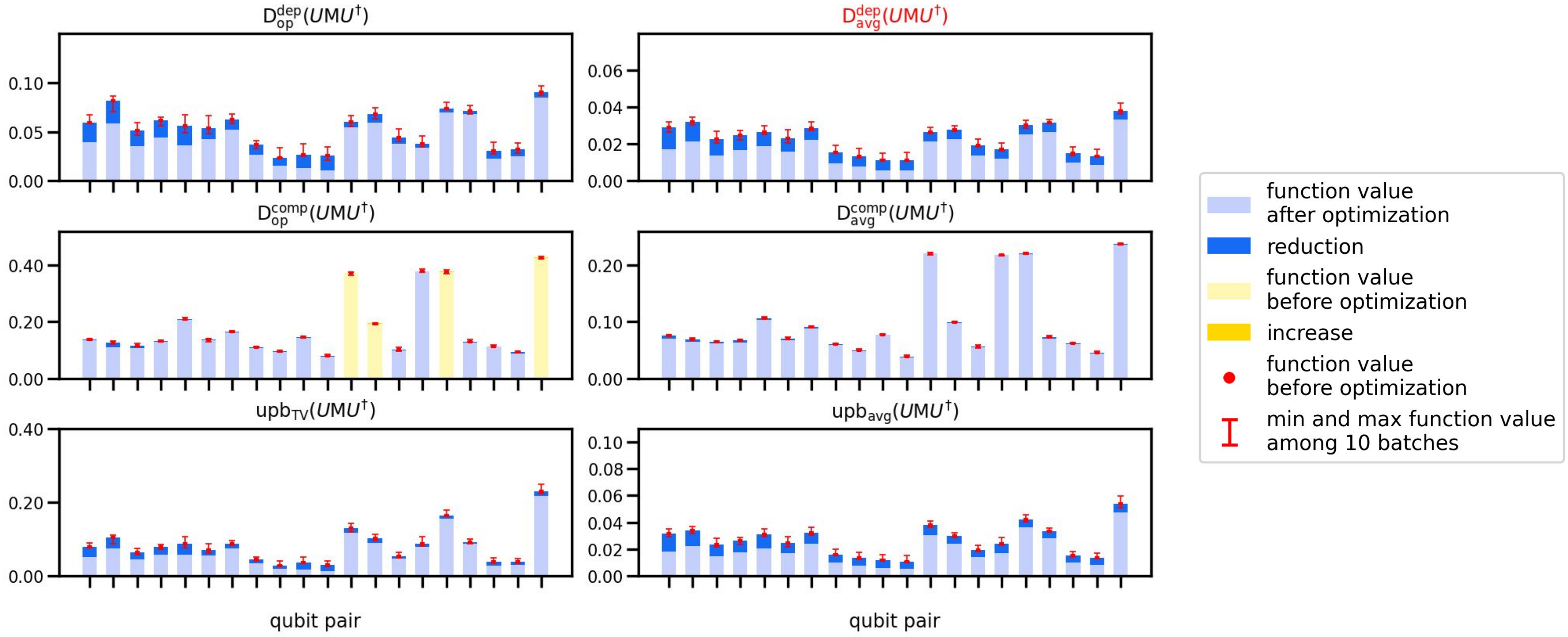

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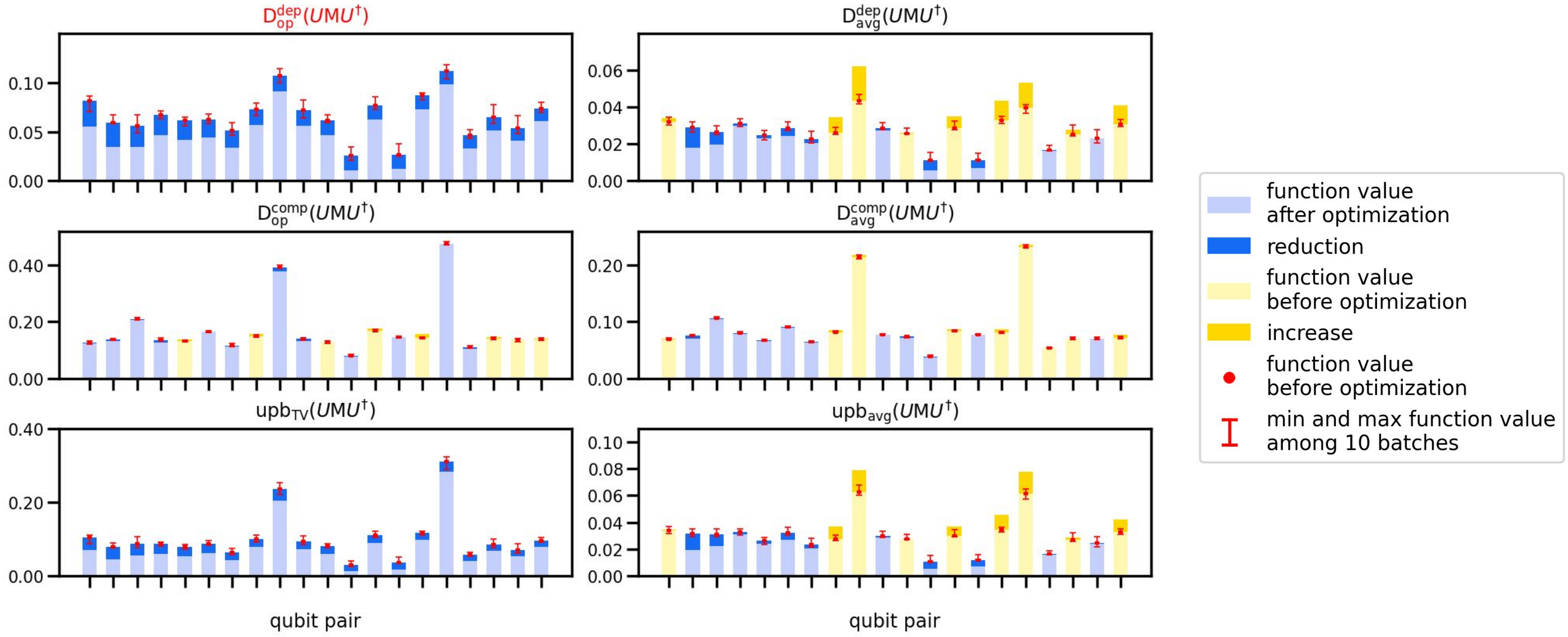

SUMMARY

- MITIGATION OF COHERENT ERRORS BY ROTATING A POVM WITH A UNITARY
- ANALYTICAL METHOD OF OPTIMIZATION OF $Off^{diag}(UMU^{+})$, $Ovl^{comp}(UMU^{+})$
- TESTS ON DATA FROM TOMOGRAPHY ON 2 DEVICES