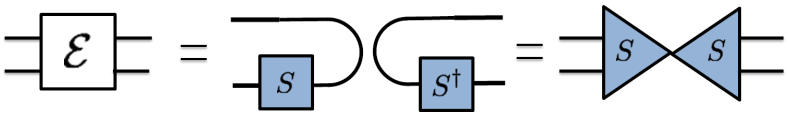

Celebrating the Choi-Jamiołkowski Isomorphism

Online Event

March 1-2, 2023

(Endorsed by the National Center for Quantum Informatics (KCIK), Gdańsk)

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Contents

Details of the Workshop

The **Choi-Jamiołkowski Isomorphism** is a remarkable result in the field of open quantum systems and quantum information theory establishing the correspondence between linear maps in operator algebras and bipartite operators in the corresponding Hilbert spaces. Nearly five decades ago it was established that positive maps correspond to block-positive operators [1] and completely positive maps correspond to positive operators [2]. The workshop aims to celebrate this fundamental result and is devoted to new frontiers in the research in open quantum systems, entanglement and quantum information theory.

[1] A. Jamiołkowski, "Linear transformations which preserve trace and positive semidefiniteness of operators", Rep. Math. Phys. 3, 275-278 (1972).

[2] M.-D. Choi, "Completely positive linear maps on complex matrices", Linear Algebra Appl. 10, 285-290 (1975).

Organizers of the Workshop

Dariusz Chruściński (Nicolaus Copernicus University, Poland)

Vinayak Jagadish (Jagiellonian University, Poland)

Francesco Petruccione (Stellenbosch University, South Africa)

Karol Życzkowski (Jagiellonian University, Poland)

Zoom Link

`https://zoom.us/j/93291227195?pwd=eGRqL2kzUlJpZG1Xd3BvUVNuMHlVQT09`

Meeting ID: 932 9122 7195

Passcode: WXeQY2

Programme

Wednesday, 1 March 2023
(Central European Time: GMT+1)

Session Chair: Karol Życzkowski

15.00-15.10 Karol Życzkowski (Kraków)
Opening Address

15.10-15.40 Guest of Honor
Prof. Andrzej Jamiołkowski (Toruń)

15.45-16.15 **Giulio Chiribella** (Hongkong)
Higher-order quantum processes and quantum causal structures

16.20-16.50 **Saverio Pascazio** (Bari)
The Choi-Jamiołkowski Isomorphism: when Maths meets Physics

16.55-17.15 —COFFEE BREAK—

Session Chair: Dariusz Chruściński

17.15-17.45 **Erling Størmer** (Oslo)
Generalizations of the Choi matrix

17.50-18.20 **Fabio Benatti** (Trieste)
A few puzzles in open quantum dynamics

18.25-18.55 **Ingemar Bengtsson** (Stockholm)
The Heisenberg groups and the dimensions of Hilbert spaces

Thursday, 2 March 2023
(Central European Time: GMT+1)

Session Chair: Francesco Petruccione

15.00-15.30 **Ángela Capel Cuevas** (Tübingen)
A generic quantum Wielandt's inequality

15.35-16.05 **Emily Adlam** (London)
The Operational Choi-Jamiołkowski Isomorphism

16.10-16.40 **Alexander Müller-Hermes** (Oslo)
Non-decomposable positive maps from tensor products

16.45-17.00 —COFFEE BREAK—

Session Chair: Vinayak Jagadish

17.00-17.30 **Sabrina Maniscalco** (Helsinki)
Emergent entanglement structures and self-similarity in quantum spin chains

17.35-18.05 **Vern Paulsen** (Waterloo)
Positive Maps and Entanglement in Real Hilbert Spaces

18.10-18.40 Guest of Honor
Prof. Man-Duen Choi (Toronto)

18.45-19.00 Francesco Petruccione (Stellenbosch)
Closing Remarks

Abstracts

The Operational Choi-Jamiołkowski Isomorphism

2 Mar
15.35-16.05

Emily Adlam

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I use an operational formulation of the Choi-Jamiołkowski isomorphism to explore an approach to quantum mechanics in which the state is not the fundamental object. I first situate this project in the context of generalized probabilistic theories and argue that this framework may be understood as a means of drawing conclusions about the intratheoretic causal structure of quantum mechanics which are independent of any specific ontological picture. I then give an operational formulation of the Choi-Jamiołkowski isomorphism and show that, in an operational theory which exhibits this isomorphism, several features of the theory which are usually regarded as properties of the quantum state can be derived from constraints on non-local correlations. This demonstrates that there is no need to postulate states to be the bearers of these properties, since they can be understood as consequences of a fundamental equivalence between multipartite and temporal correlations.

A few puzzles in open quantum dynamics

1 Mar
17.50-18.20

Fabio Benatti

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Despite its long history, research in open quantum dynamics still provides unexpected facets. We shall discuss some of them that are connected with entanglement generation and super-activation of back-flow of information by means of tensor products of dynamical maps.

The Heisenberg groups and the dimensions of Hilbert spaces

1 Mar
18.25-18.55

Ingemar Bengtsson

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Finite Heisenberg groups have a certain universal status. In every finite dimensional Hilbert space there is at least one Heisenberg group that acts irreducibly in this dimension, and in no other. I will describe some dimension dependent structures that arise in this way, and some connections between seemingly different dimensions that arise from them.

Higher-order quantum processes and quantum causal structures

1 Mar
15.45-16.15

Giulio Chiribella

QICI Quantum Information and Computation Initiative, Department of Computer Science, The University of Hong Kong, Pokfulam Road, Hong Kong
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One of the most profound insights of the Choi-Jamiołkowski isomorphism is that quantum processes can be treated as quantum states. Following this idea, it is natural to consider a kind of super-processes that transform quantum processes into quantum processes, in a similar way as ordinary processes transform quantum states into quantum states. This construction can be iterated recursively, generating an infinite hierarchy of processes of increasingly higher orders. Physically, this hierarchy corresponds to an extension of the framework of quantum circuits, including the ordinary acyclic circuits considered in quantum computing, as well as a new type of quantum circuits with cycles. In this talk I will present the main notions in the study of higher order quantum processes, discussing their application to quantum information and their connection with the study of causal structure in quantum mechanics.

A generic quantum Wielandt's inequality

2 Mar
15.00-15.30

Ángela Capel Cuevas

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In this talk, I will provide a generic version of quantum Wielandt's inequality, which gives an optimal upper bound on the minimal length such that products of that length of n-dimensional matrices in a generating system span the whole matrix algebra with probability one. I will show that this length generically is of order $\Theta(\log n)$, as opposed to the general case, in which the best bound to the date is $O(n^2 \log n)$. We will discuss the implications of this result as a new bound on the primitivity index of a random quantum channel, as well as to show that almost any translation-invariant (with periodic boundary conditions) matrix product state with length of order $\Omega(\log n)$ is the unique ground state of a local Hamiltonian. Finally, we will comment on the possibility of extending these results to Lie algebras. This is based on joint work with Yifan Jia.

Emergent entanglement structures and self-similarity in quantum spin chains

2 Mar
17.00-17.30

Sabrina Maniscalco

QTF Centre of Excellence, Department of Physics, University of Helsinki, Finland
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We introduce an experimentally accessible network representation for many-body quantum states based on entanglement between all pairs of its constituents. We illustrate the power of this representation by applying it to a paradigmatic spin chain model, the XX model, and showing that it brings to light new phenomena. The analysis of these entanglement networks reveals that the gradual establishment of quasi-long range order is accompanied by a symmetry regarding single-spin concurrence distributions, as well as by instabilities in the network topology. Moreover, we identify the existence of emergent entanglement structures, spatially localised communities enforced by the global symmetry of the system that can be revealed by model-agnostic community detection algorithms. The network representation further unveils the existence of structural classes and a cyclic self-similarity in the state, which we conjecture to be intimately linked to the community structure. Our results demonstrate that the use of tools and concepts from complex network theory enables the discovery, understanding, and description of new physical phenomena even in models studied for decades.

Non-decomposable positive maps from tensor products

2 Mar
16.10-16.40

Alexander Müller-Hermes
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Understanding how positivity of maps behaves under tensor products is linked to several open problems in quantum information theory. In my talk, I will present some recent results on how non-decomposable positive maps can arise from tensor products and tensor powers of decomposable maps. The Choi-Jamiołkowski isomorphism is an indispensable tool in this line of research.

The Choi-Jamiołkowski Isomorphism: when Maths meets Physics

1 Mar
16.20-16.50

Saverio Pascazio
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I discuss the interplay between Physics and Mathematics, from a personal perspective, in the light of the celebrated Choi-Jamiołkowski Isomorphism. I argue that it is desirable that a physical law, when expressed in terms of a differential equation, should admit any initial conditions. Choi-Jamiołkowski isomorphism, complete positivity, correlations and entanglement form a facet pattern that defines the correct formulation of the dynamics of open quantum systems. A formulation that requires coordination and teamwork if all its ingredients are to be kept in perfect alignment.

Positive Maps and Entanglement in Real Hilbert Spaces

2 Mar
17.35-18.05

Vern Paulsen
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Partially motivated by recent research in quantum physics, we take a closer look at the similarities and differences between the study of positive maps, separability, and entanglement in the real and complex case. It is possible for real matrices to be entangled as operators on a real Hilbert space and yet separable when regarded as acting on a complex space. These two distinct theories of entanglement in the real case correspond to two different theories of entanglement breaking maps in the real case. Finally, we see what these differences have to say about real versions of the PPT-squared conjecture. Based on joint research with G. Chiribella, K.R. Davidson, and M. Rahaman.

Generalizations of the Choi matrix

1 Mar
17.15-17.45

Erling Størmer
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We first study generalizations of Choi matrices for linear maps of the $n \times n$ matrices into themselves. Then we generalize this to certain maps of Von Neumann algebras into themselves.
