Genuine Bell nonlocality in many-body quantum systems

Bachelor thesis written under supervision of
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Genuine multipartite entanglement

$$\rho_{AB} = \sum_i p_i \left|\psi_A^i\middle\rangle\!\middle\langle\psi_A^i\right| \otimes \left|\phi_B^i\middle\rangle\!\middle\langle\phi_B^i\right|, \quad p_i \geqslant 0, \quad \sum_i p_i = 1$$

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GME

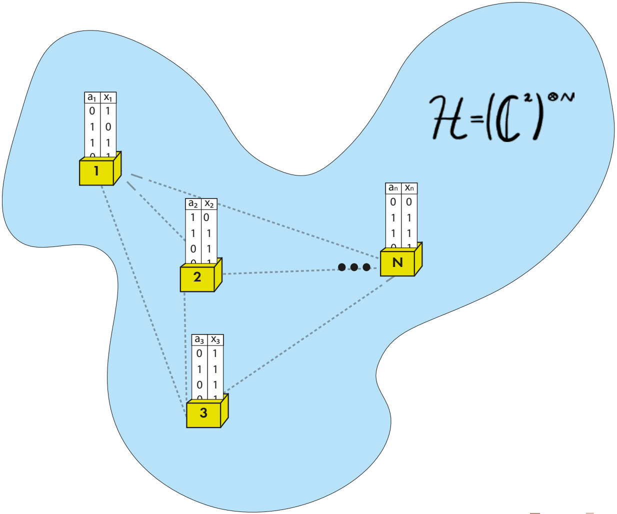
States that do not admit the above are GME states.

Bell-like experiment

In (N, m, d) scenario we obtain the conditional probabilities

$$p(a_1, ..., a_N | x_1, ... x_N),$$

$$a_i \in \{1, \ldots, d\} \qquad x_i \in \{1, \ldots, m\}.$$



Genuine multipartite nonlocality

$$p(a_1, a_2|x_1, x_2) = \sum_\lambda q(\lambda)p(a_1|x_1, \lambda)p(a_2|x_2, \lambda)$$

- Correlations bilocal across a partition $g|\bar{g}$:

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GMNL

Correlations that are not bilocal are GMNL.

Beyond the simplest scenario

Two subsystems

- There exist mixed 2-qudit entangled states that are local.

More subsystems

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  - 3-qubit states with symmetry between 2 qubits
  - special cases e.g. GHZ states for any n
  - some numerical results e.g 3-qubit states

Bell inequalities witnessing GMNL

F. J. Curchod et al., New J. Phys. 21, 023016 (2019)

- CHSH inequality

$$IAB = p(00|00) - p(01|01) - p(10|10) - p(00|11) \leqslant 0$$

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- Let us consider partition $g|\bar{g}$ RYSUNEK

Bell inequalities witnessing GMNL

Centered inequalities

$$I_1^n := \sum_{i=2}^{n} I_{\vec{0}|\vec{0}}^{1i} - (n-2)p(\vec{0}|\vec{0}) \leqslant 0$$

Rysunek

Symmetrical inequalities

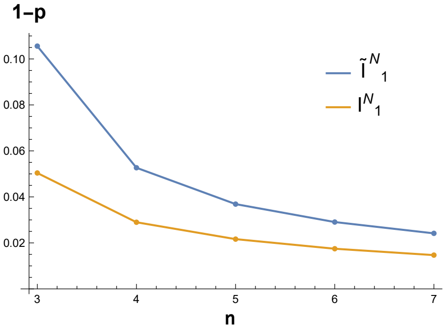
$$I_{sym}^n := \sum_i \sum_{j>i} I_{\vec{0}|\vec{0}}^{A_i A_j} - \binom{n-1}{2} p(\vec{0}|\vec{0}) \leqslant 0$$

Rysunek

Results

$$I_1^n := \sum_{i=2}^{n} I^{1i}_{\vec{0}|\vec{0}} - (n-2)p(\vec{0}|\vec{0}) \leqslant 0$$

$$\tilde{I}_1^n := \sum_{i=2}^{n} I^{1i}_{\vec{0}|\vec{0}} - (n-2)p(\vec{0}|\vec{0}) + (n-2)p(1\vec{0}|1\vec{0}) \leqslant 0$$



While in the case of 3-qubit GHZ I_1^n is resistant to around 5% of white noise, the new inequality for 10% still detects GMNL.

More general scheme of lifting allows for derivation inequalities for qudits.