# Experimental proof that nonlocal quantum entanglement is real!

John F. Clauser

817 Hawthorne Drive, Walnut Creek, CA, USA 94596
website: johnclauser.com, email: bobbi_john@jfcbat.com

# Schrödinger's (first) equation

- Almost every Quantum Mechanics textbook begins by introducing Schrödinger's (first) equation.
- It is a wave equation that describes a complex-valued "wave function", $\psi(\mathbf{r},t)$, that "propagates" in space-time.
- The absolute square, $\psi^*(\mathbf{r},t)\ \psi(\mathbf{r},t)$, describes so-called probability-density waves that similarly propagate in space-time. When it is multiplied by the particle's mass or charge, it gives the local mass or charge density of a beam of non-interacting particles. When Schrödinger's equation is applied to Hydrogen, it describes standing waves that surround the nucleus.
- It has a set of discrete solutions called eigenfunctions. They describe the allowed energy levels, i.e. the so-called and previously mysterious stationary states of Hydrogen. Each eigenfunction is associated with a different standing-wave pattern.
- The associated energy eigenvalues and their differences correspond <u>exactly</u> to the observed energy levels and spectra of Hydrogen.
- A general solution to Schrödinger's equation is given by a sum (superposition) of eigenfunctions.
- The probability or charge density for an atom in superposition of a pair of states oscillates like an electric dipole antenna radiating energy at the difference frequency of the oscillating states (the old Schrödinger or semiclassical interpretation).
- Max Born is responsible for the probability interpretation of $\psi^*\psi$.
- Max Born also first applied Green's theorem from vector calculus to Schrödinger's equation, and thereby derived a "conserved probability current" flowing in space. It, in turn, demonstrates flux conservation in Rutherford scattering.
- Schrödinger's (first) equation was an instant sensation and was quickly adopted as a law of nature.

# What is quantum entanglement?

- Schrödinger actually produced two very different equations for Quantum Mechanics – the first one describes Hydrogen (with only one electron), and a second very different one describes Helium (with two electrons).
- Schrödinger’s second equation for $N \geq 2$ multi-particle systems is usually described only in late chapters of most Quantum Mechanics textbooks.
- Schrödinger’s second equation for $N \geq 2$ multi-particle systems had unexpected solutions that introduce a new a phenomenon – quantum entanglement.
- Schrödinger invented the term “quantum entanglement” for $N \geq 2$ multi-particle systems.
- Two or more particles do not need to be identical for them to be entangled.
- The general wave-function solution for a pair of independent, not entangled, particles is a simple product of the individual wave functions of the two individual particles, with each individual-particle wave function being a sum of its eigenfunctions.
- In general the wave function for a pair of <u>entangled</u> particles is a sum of products of the wave-function eigenfunctions of the two individual particles.
- Two entangled particles exhibit intimately correlated behaviors.
- Via evolution under Schrödinger’s (second) equation, particles that have ever interacted remain entangled.
- vonNeumann, in his Quantum Mechanics textbook, asserts that the only means for terminating entanglement of a pair of particles is by the measurement of the state of either particle. Wigner, d’Espagnat, and Shimony proved that transition from a pure state to a statistical mixture (as is required by a measurement operation) requires a non-unitary evolution of the wave function, that cannot not proceed via Schrödinger (unitary) evolution.

# What is <u>non-local</u> entanglement?

- Within a Helium atom, the two electrons are always in intimate contact with each other and are continuously interacting with each other. It seems hardly surprising that their individual behaviors are intimately correlated.
- Einstein and Schrödinger independently noticed a peculiar feature of quantum entanglement.  Two (or more) entangled particles remain intimately correlated in their behavior, no matter how far apart they are.
- Einstein and Schrödinger were both deeply disturbed by this new seemingly bizarre prediction made by $N > 1$ particle Quantum Mechanics (Schrödinger's $2^{nd}$ Eqn.).
- Einstein, Podolsky, and Rosen (EPR, 1935) proposed that Quantum Mechanics is an incomplete theory an requires the addition of additional hidden variables to explain <u>non-local</u> quantum entanglement.
- Following EPR, Schrödinger and Furry independently hypothesized that entanglement perhaps disappears when the particles are widely separated. But since their hypothesis changed the predictions of the theory, it was dismissed summarily .
- Nils Bohr alternatively argued that hidden variables are unnecessary, depending on how one viewed the problem. The EPR – Bohr differences were never fully resolved, and are echoed in different viewpoints exhibited between treatments in various textbooks.

# Experimental Need for Entanglement

- Prior to CHSH 1969, the observed spectrum of Helium (and other multi-electron atoms and molecules) was overwhelmingly sufficient to convince nearly everyone that <u>local</u> entanglement is a correct and necessary component of Quantum Mechanics.
- Prior to CHSH 1969, nobody suggested <u>actually doing</u> an experiment to actually test the bizarre prediction of <u>non-local</u> entanglement made by Quantum Mechanics.
- The only exception was by Bohm and Aharonov (1957), who showed that the Wu-Shaknov (1950) experiment refuted the hypothesis by Schrödinger and Furry suggesting that the particles behaved independently at large separations.

# What is a Bell inequality? – There are three of them

- #1 John Bell’s 1964 inequality
  - **It is heuristically of <u>immense</u> importance. It showed that EPR’s hidden-variable argument leads to the opposite conclusion that EPR came to!**
  - Einstein, Schrödinger, Bohr, <u>and Bell</u> automatically assumed that Quantum Mechanics <u>always</u> gives correct predictions.
  - No experimentally testable prediction was offered by Bell 1964, or was even possible given his restrictive assumptions.
- #2 Clauser-Horne-Shimony-Holt “CHSH” 1969 inequality
  - **Unlike Einstein, Schrödinger, Bohr, and Bell, CHSH <u>did not</u> automatically assume that Quantum Mechanics always gives correct predictions. (Heresy!)**
  - **A detailed experiment was correspondingly proposed by CHSH to test the two different predictions.**
  - CHSH provided an inequality that gives an actual experimental prediction for that experiment, <u>that is different from the Quantum Mechanical prediction</u>.
- #3 Clauser-Horne “CH” 1974 inequality and Local Realism
  - Clauser and Horne significantly generalized EPR’s hidden-variable theories to cover Einstein’s whole platform for doing physics. The resulting very general theory is now called Local Realism.
  - Clauser-Horne produced a new more general inequality that covers experiments with a much simpler simpler experimental layout. **It tests all theories consistent with the Local Realism framework.**
  - The CH inequality gives the first loophole-free experimental prediction.
  - The CH inequality has been tested experimentally. Local Realism is dead (Jim).

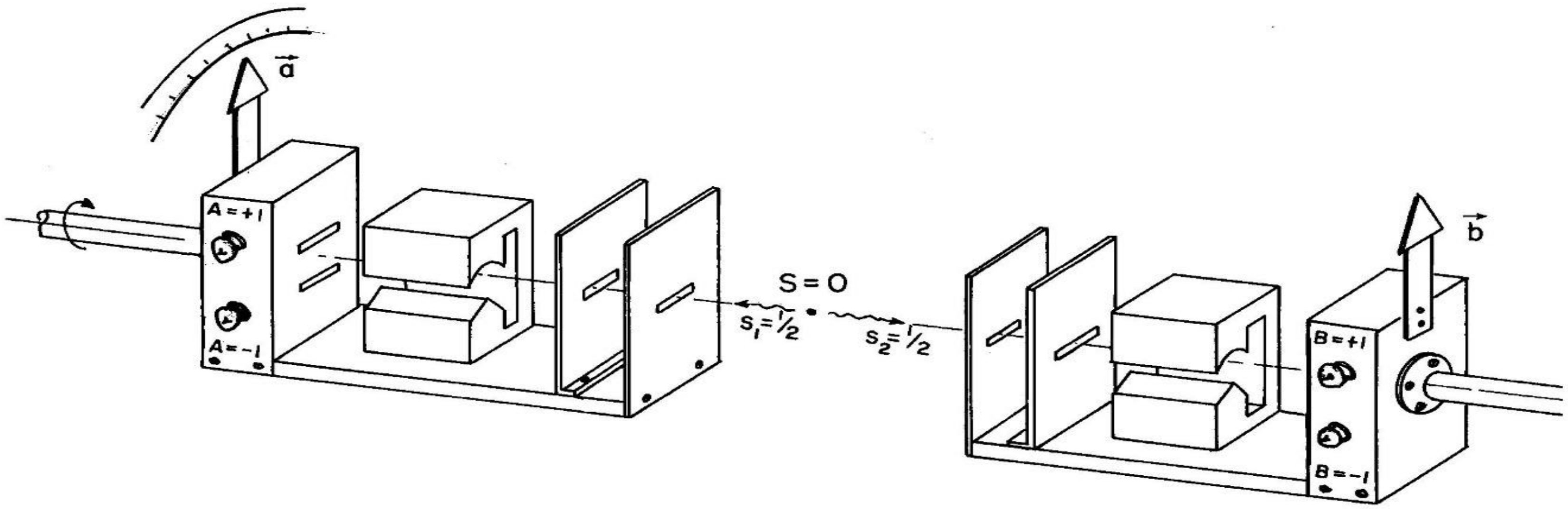
# Did I mention heresy?

- At the time I was proposing and performing the first Bell – inequality tests, it was politically incorrect within the physics community to question or critique the foundations, prevailing interpretation, or experimental predictions of Quantum Mechanics.
- At the time I was proposing the first Bell inequality test, I was still a graduate student at Columbia University. My thesis advisor thought that a Bell inequality test was a bad idea. He directed me to talk to Columbia theorist, Robert Serber, who was famous for authoring the design "primer" for the Los Alamos atomic bomb. Serber summarily dismissed me from his office saying "That is an interesting idea perhaps worth a *Note to the Editor*, but no descent experimentalist would ever go to the effort to actually do such an experiment."
- Virtually everyone on my graduate faculty at Columbia similarly told me that I would ruin my career by doing experimental tests of Quantum Mechanics. They all advised me not to proceed. Many were subsequently outraged by my publication of CHSH, which I did without seeking their permission. My CHSH coauthor, Dick Holt, got a similar reaction from his physics faculty at Harvard.
- While I was performing the 1972 Freedman Clauser experiment at UC Berkeley, I went to discuss the experiment with Richard Feynman at my undergraduate alma mater, Caltech. He was outraged and offended by my heresy and threw me out of his office. So doing, he adamantly and indignantly proclaimed "Quantum Mechanics needs no further testing."
- Fifty years later, the Nobel Prize was the reward for my heresy.

# Bell inequality #1: John Bell’s 1964 inequality – with no experimentally testable prediction

- Like Einstein, Schrödinger and Bohr, Bell similarly assumes that Quantum Mechanics always gives correct predictions. Correspondingly, no actual experiment is proposed or cited by Bell.
- Bell considers the predictions for the correlation of measurement results performed on separated entangled particles, specifically for the entangled spin state of a pair of spin-½ atoms in Bohm’s 1950 *gedankenexperiment* example of the EPR paradox.
- John Bell (1964), with great irony, showed that EPR’s argument led to the opposite conclusion that EPR came to.
- Bell showed that EPR’s discussion of entanglement implied an incompatibility between the predictions of Quantum Mechanics and EPR’s Local Hidden Variable Theories for the predicted correlation of the spin measurements.
- Bell showed that the intimately correlated behaviors of two entangled particles governed by Quantum Mechanics cannot be explained via the introduction of hidden variables.
- Bell’s 1964 inequality unfortunately requires an impossible-to-achieve condition, and is thus untestable. Its proof requires that a perfect correlation must occur for at least one pair of relative apparatus-parameter settings. It thus applies only to situations and configurations that can never exist in reality. Bell’s 1964 inequality thus has only heuristic value. It cannot be applied to a real experiment.

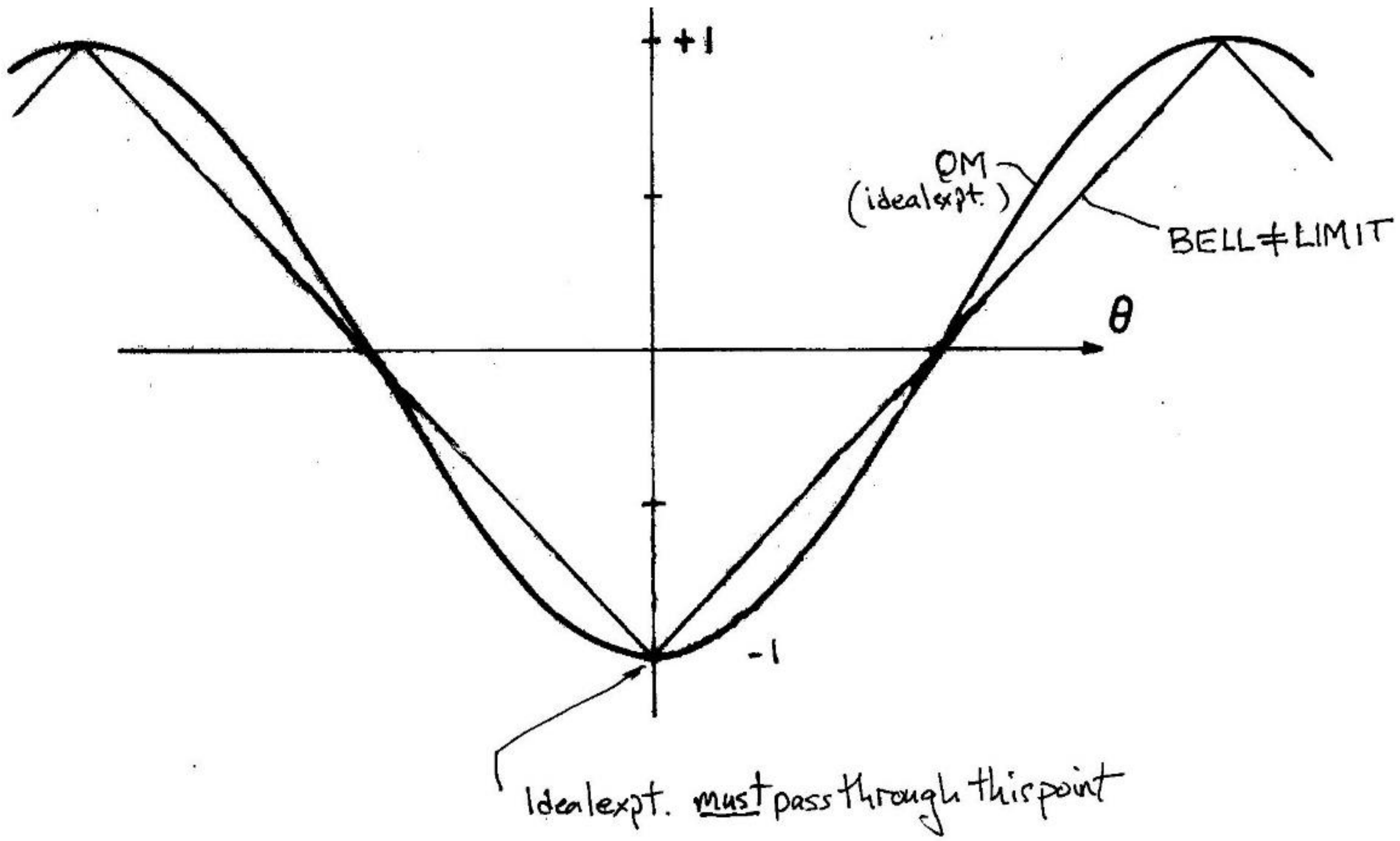
# Bohm’s 1950 *Gedankenexperiment*




$$\text{Q.M.} \Longrightarrow P(\vec{a}, \vec{b}) = \overline{AB} = \langle S=0 \mid \vec{\sigma_1}\cdot\vec{a}\ \vec{\sigma_2}\cdot\vec{b} \mid S=0 \rangle = -\vec{a}\cdot\vec{b}$$

# Bell Inequality #1's Prediction for an edeal experiment –

<AB> as a function of the angle, ϴ, between the Stern-Gerlach analyzers.



# Bell inequality #2: Clauser-Horne-Shimony-Holt inequality CHSH 1969 inequality – the first experimentally testable prediction

- [J.F Clauser, M. A. Horne, A. Shimony, and R. A. Holt, *Proposed experiment to test local hidden variable theories*, Physical Review Letters **23**, 880-884 (1969).]
- **Unlike Einstein, Schrödinger, Bohr, and Bell, CHSH did not automatically assume that Quantum Mechanics always gives correct predictions. (Heresy!)**
- Inspired by Bell (1964), John Clauser, Mike Horne, Abner Shimony, and Dick Holt (CHSH, 1969) showed that Einstein-Podolsky and Rosen's (EPR) proposed Local Hidden Variable Theories are all testable with a single realizable experiment.
- CHSH designed that experiment and defined the necessary details for it.
- CHSH formulated the CHSH inequality -- the first experimentally testable "Bell inequality". The CHSH inequality is an experimental prediction made by all Local Hidden Variable Theories for that experiment.
- **CHSH's formulation does not presuppose Bell's impossible-to-achieve conditions**.
- Local Hidden Variable Theories and Quantum Mechanics make different predictions for that experiment. Thus, either theory can be refuted by the experimental results.
- We now have two theories with different predictions. Let's do an experiment to see which on is right. (Let's commit heresy!)
- CHSH identify the "detector efficiency loophole" in the application of the CHSH inequality to the results.
- CHSH provide a supplementary assumption and important experimental protocols to address this loophole. CH (1974) subsequently provide a much weaker supplementary assumption.
- Great care must be exercised in its use to preserve correct normalization of measured probability values.

# Bell inequality #3: Clauser-Horne 1974 inequality – the first loophole-free experimentally testable prediction – for Local Realism

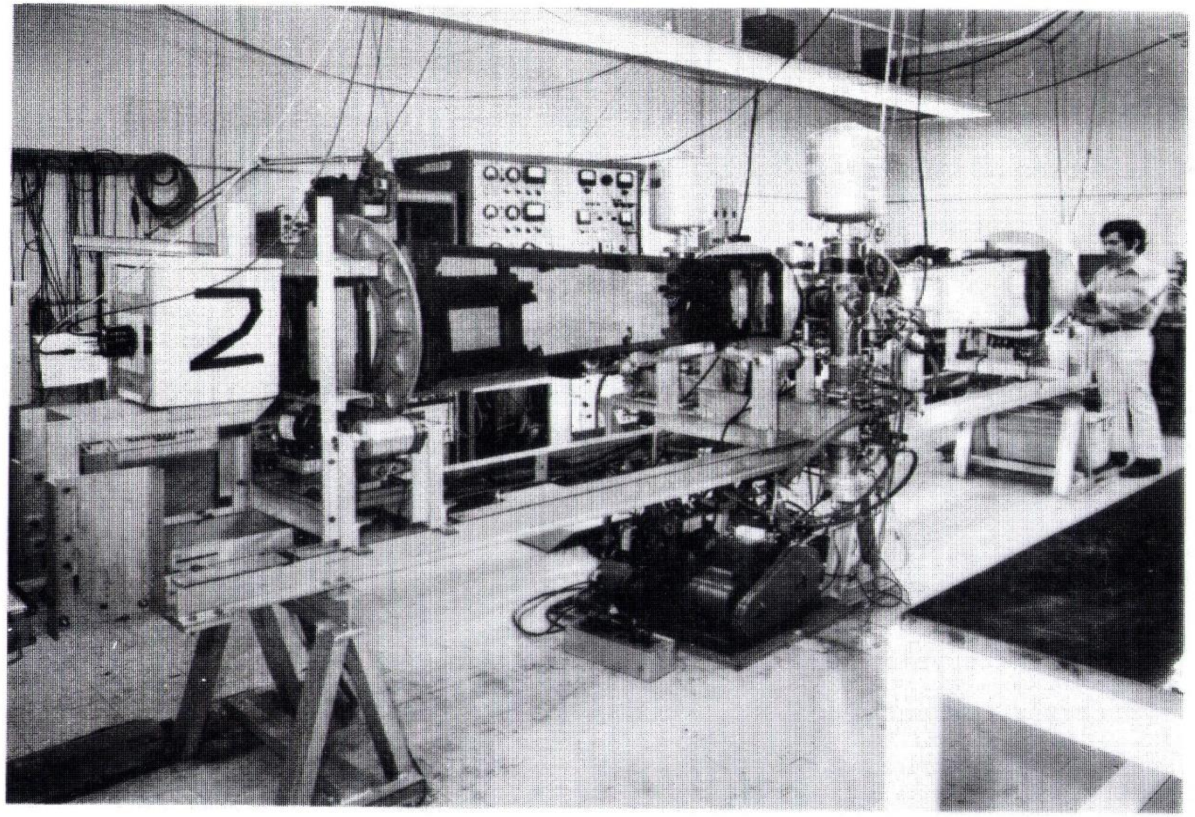
- In order to carefully define what theories quantum entanglement experiments actually refute, John Clauser and Mike Horne invented Local Realism (CH, 1974).
- <u>Local Realism is an extremely general and inclusive theory. Indeed, it underlies virtually all of Einstein's space-time platform for formulating physical theory.</u>
- Local Realism is a significant generalization of EPR's Local Hidden Variable Theories.
- For these theories, CH formulated the Clauser-Horne (CH) inequality as a loophole-free experimental prediction.
- The CH inequality allows loophole-free tests of Local Realism with no auxiliary assumptions.
- The CH inequality reduces to the CHSH inequality given very mild auxiliary assumptions, under which the Freedman and Clauser (1972) and Clauser (1976), experiments refute both Local Hidden Variable Theories and Local Realism.
- Recent (2013, 2015) experiments conclusively refute Local Realism using the CH inequality, with no required auxiliary assumptions (i.e. with no loopholes). Correspondingly, Einstein's whole platform for physical theory is untenable.

# Four experiments by John Clauser testing Local Realism and Einstein's 1917 proposed existence of localized photons

- Stuart Freedman and John Clauser (1972) performed and published the first experimental test of Local Hidden Variable Theories, and thereby first demonstrated that non-local quantum entanglement is real, and that EPR's proposed whole class of theories is untenable. <u>This experiment was the first observed violation of a Bell inequality.</u>
- John Clauser (1976) performed the second (published) experimental test of Local Hidden Variable Theories, thereby confirming the Freedman - Clauser (1972) results.
- John Clauser (1976) also tested the circular polarization correlation of entangled photon pairs.
- John Clauser (1974) first conclusively experimentally demonstrated that photons have a particle-like behavior in an experiment originally suggested by Schrödinger, but never confirmed. It was also the first observation of sub-poissonian statistics for light.

# The first published experimental test of Local Realism and Local Hidden Variable Theories, and the first observed violation of a Bell inequality.

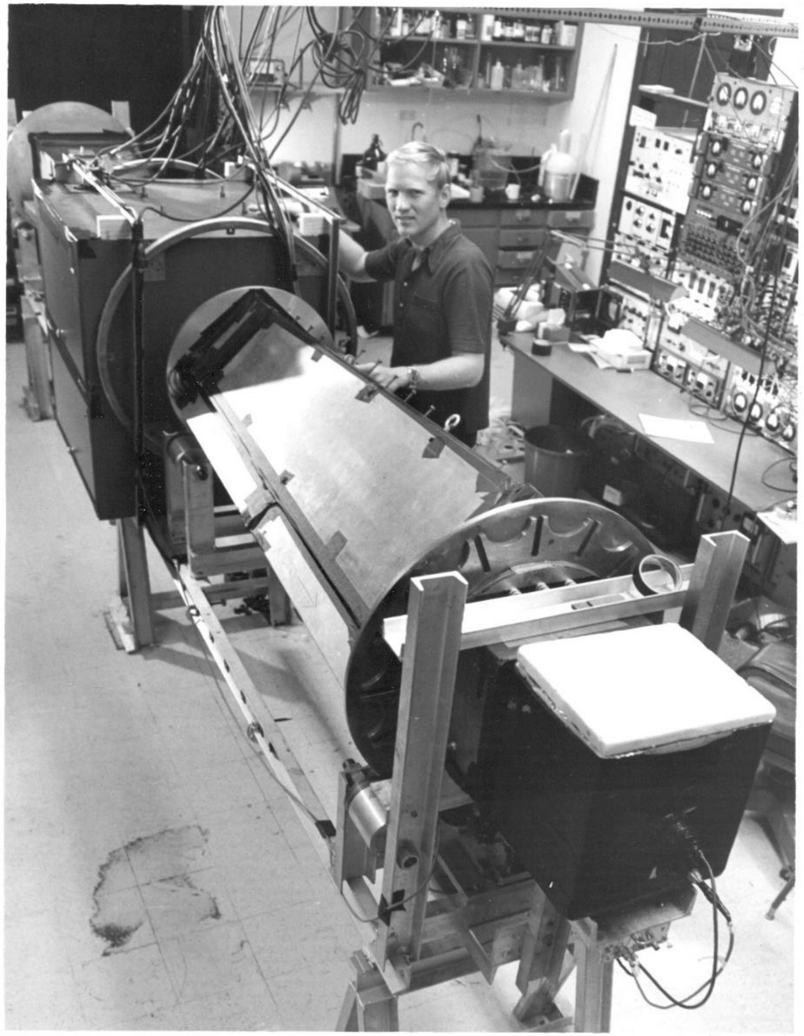
- [S. J. Freedman and J. F. Clauser, *Experimental test of local hidden variable theories*, Physical Review Letters **28**, 938 (1972).]
- The experiment was also Stuart Freedman's PhD thesis project at Univ. of Calif. - Berkeley. (Stuart is standing beside the experiment in the photograph.)
- The experiment measured the linear polarization correlation of photon pairs emitted in a 3-level cascade in Calcium.
- Excitation of Ca in an atomic beam was via UV light from a Deuterium discharge lamp. (Tunable lasers were not available in 1972.)
- The results violated the CHSH inequality by 6.3σ with no background subtraction.
- The experiment used the CHSH polarizer-removal protocol to bypass the detector efficiency loophole.



XBB 7112-5794

# The second published experimental test of Local Realism and Local Hidden Variable Theories

- [J. F. Clauser, *Experimental investigation of a polarization correlation anomaly,* Physical Review Letters **36**, 1223 (1976).]
- This was a repeat of the R.A. Holt and F. M. Pipkin experiment (Harvard U., unpublished).
- The experiment measured the linear polarization correlation of photon pairs emitted in a 3-level cascade in $^{202}$Hg.
- Excitation of Hg was via electron bombardment.
- Results violated the CHSH inequality by 4.1σ.
- Experiment used the CHSH polarizer-removal protocol to avoid detector efficiency loophole.
- The circular polarization correlation was also measured with this apparatus. [J,F. Clauser, Il Nuovo Cimento **33B**, 740 (1976)].



# Schrödinger's proposed experiment to test for a particle-like character for photons

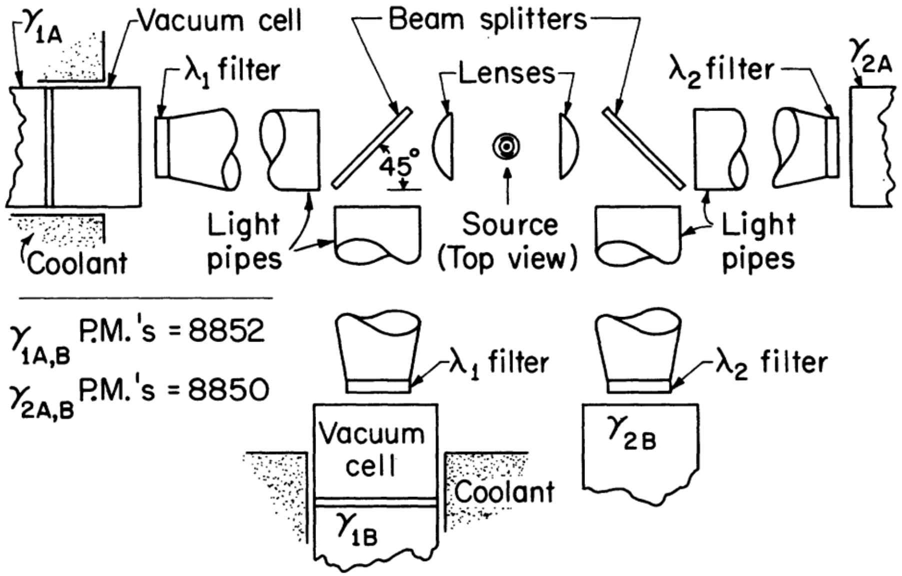
- Einstein wrote a seminal paper in 1917 about the quantum theory of the radiation field and its interaction with a gas of molecules. In this famous paper he introduced his A and B coefficients for energy transfer between the molecules and the radiation.
- He showed that in order for thermal equilibrium to obtain between the molecules and the radiation, localized quantized "photons" are emitted and absorbed by atoms, and they must behave as randomly directed particle-like "*directional energy bundles*".
- The physical nature of these "*directional energy bundles*" was unclear. Oseen offered a non-particle example of randomly directed "needle-radiation" pulse solutions to Maxwell's equations that are not particle-like, but are still highly directional.
- A particle-like behavior is required for microscopic energy conservation in Einstein's theory. However, the Bohr, Kramers, Slater theory and the "old Schrödinger interpretation" (an oscillating charge distribution acting like a dipole antenna) both conserve energy only on the average. These are also now also called semi-classical radiation theories and are still commonly taught in many simplified quantum-mechanics textbooks.
- Experimental evidence was clearly needed to establish a particle-like behavior and a wave-particle duality for photons.
- Schrödinger (1927) noticed that particle-like photons will be <u>either</u> transmitted <u>or</u> reflected at a half-silvered mirror (beam splitter). By contrast,  classical wave packets (short directional classical electromagnetic pulses of light) will be both transmitted <u>and</u> reflected by a beam splitter.
- To unambiguously test for an particle-like character for photons, Schrödinger proposed putting two phototubes in the reflected and transmitted beams behind a half-silvered mirror, and illuminating the mirror with single photons. The phototubes will sometimes record a coincidence count if a single photon behaves like a short wave-like pulse (a.k.a. wave packet), but will never see a coincidence count if it behaves like a particle.
- Ádám-Jánossy-Varga (1955) performed Schrödinger's proposed experiment, but did not use a single-photon source. Also, they grossly overestimated the effective photon-pair detection efficiency by leaving out the solid-angle loss. Their experiment was thus inconclusive (by about a factor of »$10^6$, and was unable to distinguish random accidental coincidences from "true" anomalous coincidences.

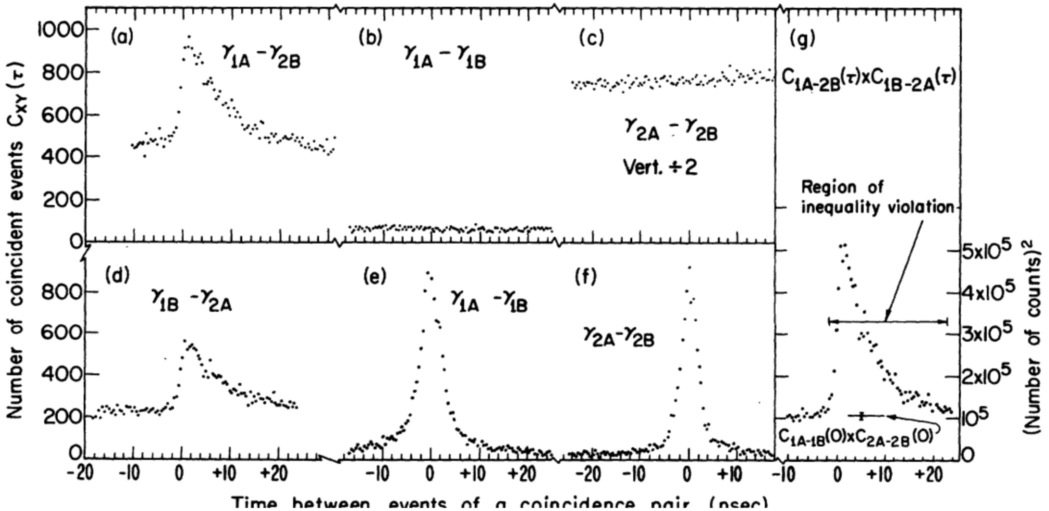
# Clauser’s (1974) experiment

- [J. F. Clauser (1974), *Experimental distinction between the quantum and classical field-theoretic predictions for the photoelectric effects*, Physical Review, **9**, 853-860.]
- Clauser’s (1974) experiment is the first <u>conclusive</u> realization of Schrödinger’s proposal. Photons $\gamma_1$ and $\gamma_2$ , are successively emitted by a two-photon cascade decay of Hg. They each respectively propagate through <u>two</u> half-silvered mirrors and then impinge on four photomultiplier tubes, $\gamma_{1A}$, $\gamma_{1B}$, $\gamma_{2A}$, and $\gamma_{2B}$ .
- The Titulaer-Glauber formalism for randomly fluctuating classical EM fields, along with the Cauchy-Schwarz inequality, require that four coincidence count rates are constrained (for “classical” wave-like photons) by

$$C_{1A\text{-}1B}(0)\, C_{2A\text{-}2B}(0) \geq C_{1A\text{-}2B}(\tau)\, C_{1B\text{-}2A}(\tau).$$

- No anomalous coincidence-counts are observed for single-photon emissions passing through a half-silvered mirror. The data overwhelmingly violate this inequality, and thereby provide the first observation of sub-poisonian statistics for light. This experiment then provides the first conclusive observation of a particle-like behavior for single photons.



# Clauser-Horne-Shimony-Bell Local Realism

- In order to carefully define what quantum entanglement experiments actually prove, John Clauser and Mike Horne (1974) invented **Local Realism**, (originally called **Objective Local Theories**)**.**
- [J.F. Clauser and M.A. Horne, *Experimental consequences of objective local theories*, Phys. Rev. D **10**, 526 (1974). Refinements to the theory were given by John Bell, Abner Shimony, M.. A. Horne, and J. F. Clauser (1976,1978), in *An exchange on Local Beables*, Dialectica, **9**, 85-110. Further refinements to the theory were by J.F. Clauser and A. Shimony, (1978) *Bell's theorem: experimental tests and implications,* Rep. Prog. Phys., **41**, 1881], who renamed **Objective Local Theories** to **Local Realism**.]
- **Local Realism** is a very general theory, and is a significant generalization of EPR's **Local Hidden Variable Theories**. Local Realism is <u>the basic foundation and framework for all of Einstein's physical theory</u> (including General Relativity).
- Unfortunately, <u>**Local Realism** was a very short lived viable theory</u>. It was already refuted by existing experiments <u>before</u> it was fully formulated. Indeed, The Freedman and Clauser (1972) and Clauser (1976), experiments refute both **Local Hidden Variable Theories** as well as **Local Realism,** given very mild auxiliary assumptions.
- Even though **Local Realism** was *dead-on arrival*, that fact wasn't good enough for many people. It thus fostered very many additional subsequent experimental tests *to totally destroy it* and also to close the CHSH loopholes. The CH inequality has only recently (2013, 2015) been tested (*and killed*) experimentally.

# Some important features of Local Realism

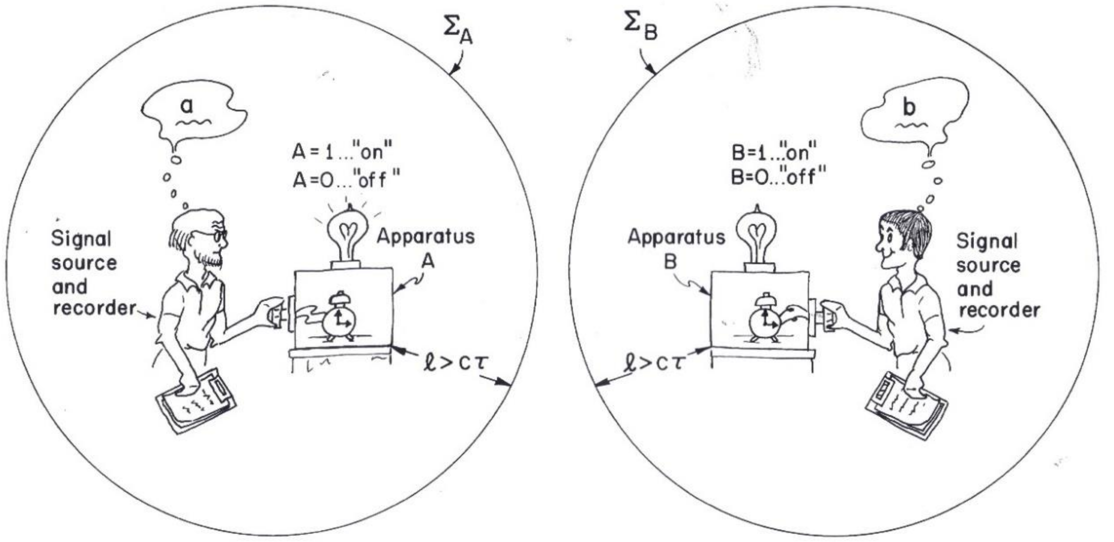
- Local Realism is extremely simple and reasonable. Anton Zeilinger once noted that if Bell's Theorem had been discovered and Local Realism formulated prior to the discovery of Quantum Mechanics, then Local Realism and the CH inequality would have been adopted as new "laws of nature".
- Clauser-Horne (1974) first formulated Local Realism along with its important new experimental prediction -- the Clauser-Horne (CH) inequality. The CH inequality allows loophole-free tests of **Local Realism** with no auxiliary assumptions.
- Causality, explicitly invoked by Bell (1964) and CHSH (1969), is neither needed, nor required. Stochastic evolution is specifically allowed.
- A requirement for particle-like behavior for a system (needed by the CHSH auxiliary assumption and tested by Clauser's 1974 experiment) is eliminated. Particles are not needed. Only experimentally observed approximately coincidental detector "clicks" are used in the theory's formulation.

# What does Local Realism assume?

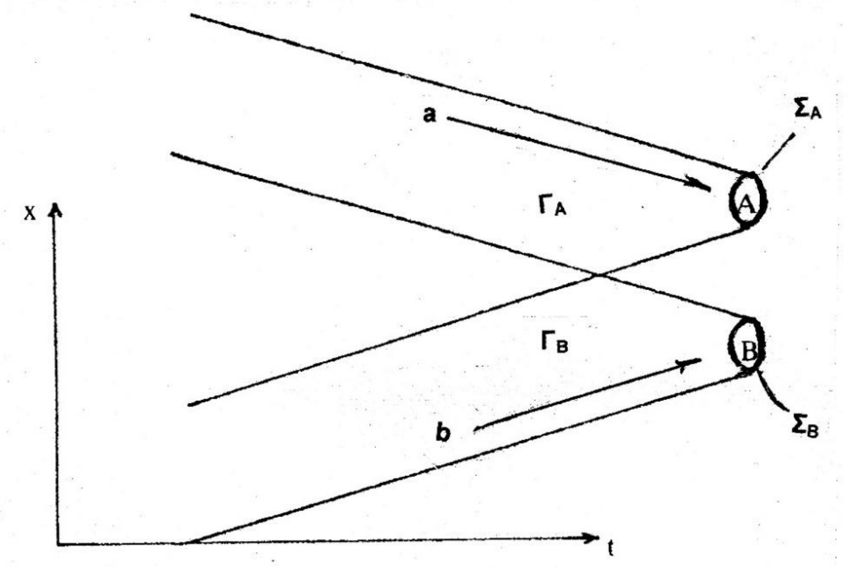
- Local Realism assumes that nature consists of stuff, i.e. of everyday "matter", of objectively-real objects, of stuff with intrinsic properties. We may not know what the stuff is, but, whatever it is, we assume that it exists and that it is distributed throughout space.
- EPR called the stuff "elements of reality". John Bell (1976) called it Local Beables. Cosmologists simply call it matter. The Bible assumes that it exists and calls it light mingled with firmament. CH called the stuff "objects" or "objective reality".
- Under CH Local Realism, objects are stuff that you can "localize" and put inside a "box". A "box" here is an imaginary closed (Gaussian) surface defining separated inside and outside volumes in laboratory space.
- Local Realism simply assumes that matter/stuff/everyday-objects/firmament exists, whether or not we observe it/them.
- Einstein's General Relativity is a special case of Local Realism.
- General Relativity further assumes that stuff/matter has a finite mass/energy density. Local Realism is much more general and doesn't care.
- General Relativity also assumes determinism. By contrast, stuff/matter may evolve under Local Realism either deterministically or stochastically. It's OK for God to play dice for the evolution of stuff.
- Local Realism assumes that stuff within a box has intrinsic definite properties, and that when someone performs an experiment within the box, the probability of any result that obtains is somehow influenced by the properties of the stuff within that box.
- If one performs say a different experiment on the same stuff with say different experimental parameters, then presumably a different probability of the result applies.
- Definite results are assumed to obtain for experiments performed inside the box, whether or not we look at them. Here we eliminate Bohr's mysticism. Also, c.f. Schrödinger's *reductio ad absurdum* argument: Local Realism simply assumes that Schrödinger's cat is either alive or dead before we look at it. We just don't know which.

# What is a “box?” A “box” here is an imaginary closed (Gaussian) surface, that defines separated inside and outside volumes. Boxes contain stuff. Figures show a pair of space-like separated 4D “boxes”

The Figure below shows a snapshot x-y diagram of a pair of space-like separated 3D “boxes”, 3D boxes, labeled $\sum_A$ and $\sum_B$ . Both are shrinking at light-speed. The two boxes surround two separated apparatuses. Each apparatus measures a selected property of the “stuff” contained within the box, i.e. it measures properties of an object within the box. The 2 men inside the boxes are now called Alice and Bob.




The Figure below shows an x-t diagram of the same two “boxes”, giving us now two backward-light-cone-shaped 4D boxes. Any correlation of results of experiments at A and B is presumably due to communal stuff located in the overlap region of the two backward-light-cone-shaped boxes.



# What does Local Realism predict for experiments performed inside the two boxes?

- Suppose that one has two widely separated boxes, each containing stuff/matter. Local Realism further assumes that the experimental parameter choice made in one box cannot affect the experimental outcome in the distant box. (Einstein locality).
- Local Realism thereby prohibits Einstein's spooky action-at-a-distance. It enforces Einstein's causality and prohibits any such nonlocal cause and effect.
- Surprisingly, these simple and very reasonable assumptions are sufficient *on their own* to allow derivation of a second important experimental prediction limiting the correlation between experimental results obtained in the separated boxes. That prediction is the 1974 Clauser–Horne (CH) inequality.

# Local Realism and the CH inequality have now been recently tested and convincingly refuted

## It's dead, Jim! (with my apologies to Star Trek)

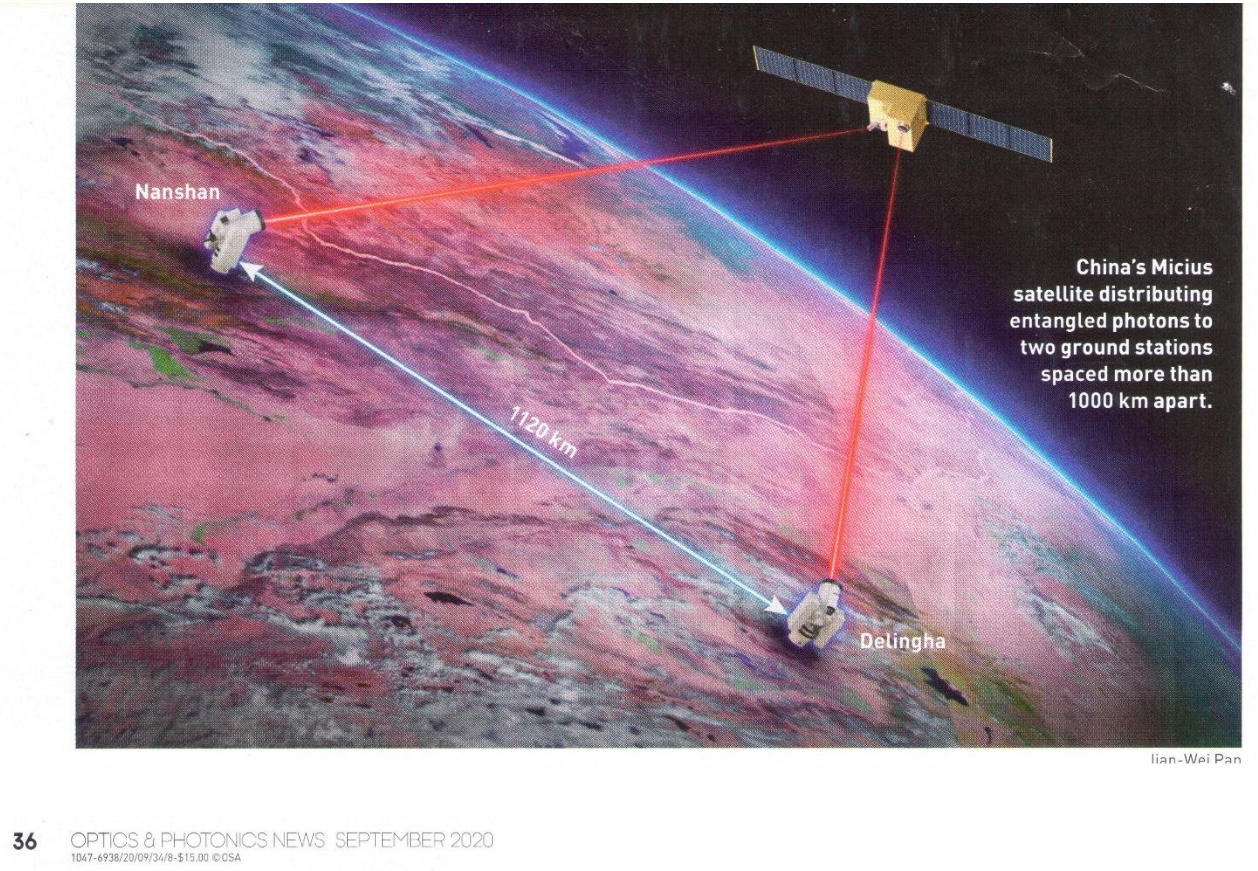
- Testing Local Realism and the CH inequality was considered by many researchers to be sufficiently important, both to eliminate the CHSH loopholes and to disprove **Local Realism**. Considerable effort was thus marshaled, as quantum optics technology improved and permitted.
- The technology for testing the CH inequality was not available in 1974. It later became a major challenge for experimentalists to do so.
- Violation of the CH inequality was first achieved in 2013 for boxes that were widely but not space-like separated by
  - M. Giustina, *et al.,* (2013) Nature **497**, 227-230. (Anton Zeilinger's group)
  - B. G, Christensen, *et al.,* (2013) Phys. Rev. Lett. **111**, 130406. (Paul Kwiat's group)
- Violation of the CH inequality was first achieved in 2015 for space-like separated boxes by
  - M. Giustina, *et al.,* (2015) Phys. Rev. Lett. 115, 250401. (Anton Zeilinger group)
  - L. K. Shalm, *et al*., (2015) Phys. Rev. Lett. 115, 250402. (Paul Kwiat's group)
- Einstein's basic platform for a space-time description of physics has been hereby destroyed. **It's dead, Jim!**

# An important application for nonlocal quantum entanglement: Can you always put a single “bit” in a box?

- The simplest possible “object” definable under **Local Realism** is a single bit of information.
- A quantum bit is called a qubit. It may or may not be a classical “object”.
- The CH inequality, when violated by such an entity, shows that quantum mechanical entities exist that cannot be localized in a single box.
- Indeed, such an entity (a qubit) may be distributed into two (or more) widely separated boxes, and measurements must be made in both boxes in order to determine the qubit’s value.
- The refutation of Local Realism for such objects thus provides the basis for quantum information and quantum cryptography.

# Quantum key distribution - an application of non-local quantum entanglement.

- The Chinese Micius satellite (Jain-Wei Pan) uses a configuration that is the same as that used by the Freedman-Clauser (1972) experiment. Entangled photon pairs are projected from the satellite to two ground stations that measure the polarization correlation of the pairs.
- Random polarizer orientations are used at the ground stations, and coincidence counts are measured.
- Temporally, the configuration is the same as Alice and Bob's two separated "boxes" (4D Gaussian surfaces) shown above.
- Bennet and Brassard (1984) showed that these measurements, coupled with openly-public (hackable) ground communication, can provide secure encrypted communication between the measurement apparatuses in the ground stations.
- CHSH inequality violation for some of the pairs is used to verify the absence of eavesdropping.



# Born's Ambiguity: Configuration space vs Lab Space? – An ignored "elephant in the room".

- Recall that Schrödinger produced two very different equations – one that describes Hydrogen (with only one electron), and a very different one that describes Helium (with two electrons). His second equation had unexpected solutions with quantum entanglement. Entanglement is not present in first equation.
- <u>Nobody has noticed (until now*) the fact that these two equations and their solutions are formulated in two very different "spaces". ← The "elephant"!</u>
- <u>The spaces are improperly linked by what I call Born's Ambiguity. ← The "elephant"!</u>
- This "elephant" has been hiding in the room in plain sight for more than eighty years. (Pardon the metaphor.)
- *[J. F. Clauser (2021) *Laboratory - Space and Configuration - Space Formulations of Quantum Mechanics, Versus Bell-Clauser-Horne-Shimony Local Realism, Versus Born's Ambiguity*. Chapter 3 in *Quantum Arrangements*, G. Jaeger *et al*. (eds.) Springer-Nature, pp, 31-95. -- Contributions in honor of Michael Horne]

# An inconsistent elephant that no one has noticed:

- Mathematicians (and theoretical physicists) have done us a major disservice, and have multiply defined the word “space”, while referring to things like… the space explored by Star Trek’s Enterprise, the 3-dimensional space in which we live (a. k. a. Lab space), a vector space (one set of complex numbers that somehow depends on another set of complex numbers, a. k. a. a function), and the argument-space of a function. Careless lack of specificity as to which kind of space is being used creates ambiguities and inconsistencies.
- In what space do various kinds of “waves” propagate?
  - Real valued electromagnetic waves, $\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0 \sin(\mathbf{k}\cdot\mathbf{r} - \omega t + \phi)$ propagate in Lab space.
  - Real valued sound waves (pressure and density waves), ocean waves, and Einstein’s gravitational waves all propagate in Lab space.
  - Here, $(\mathbf{r},t)$ is the argument space of the function, $\mathbf{E}(\mathbf{r},t)$.
  - Electron deBroglie waves in Hydrogen, $\psi_{lab}(\mathbf{r}_{lab},t)$ propagate in lab space, (according to the quantum mechanics books by Schiff, Merzbacher, Dicke and Wittke, French and Taylor, Feynman, etc. i.e. school #1).
  - Electron deBroglie waves in Helium, $\psi_{config}(\mathbf{r}_{1,config}, s_{1,config}, \mathbf{r}_{2,config}, s_{2,config}, t)$ propagate in configuration space. (Dicke and Wittke and Merzbacher don’t consider this “wave propagation in the normal sense.)
  - De-excitation of single and 2-photon excitations of the electromagnetic field (via vonNeumann wave-function collapse) travels via waves in Lab space with velocity » c. (See measurements by Gisin.)
  - The electric field, Helium and Hydrogen and gravity are all inconsistently treated!
  - Captain Kirk does not explore configuration space.

# The “elephant” is revealed!

- The EPR – Bohr debate has fostered two very different schools of thought. I assert that it was never really settled! (I provide an alternative to Bohr and Einstein’s discussion.)
- These two schools of thought persist today. Surprisingly, they coexist today without anyone (until now) noticing this fact (an elephant hiding in plain sight).
- CH Local Realism and quantum entanglement now bring to light, identify and contrast these two schools.
- Surprisingly, they describe two different incompatible formulations of quantum mechanics.
- School #1 is actually a form of Local Realism. It is therefore disallowed by experiment! Importantly, it cannot be generalized to describe N > 1 particle entangled systems.
- The two schools are presented inconsistently by two different sets of (pre-1980) textbook authors.

# School of thought #1:
# Einstein, Schrödinger, and Born's school of thought

The laboratory-space formulation of quantum mechanics is taught in the following (pre-1980) standard quantum mechanics textbooks

- Born (1933,1935), *Moderne Physik;* Born (1969, English translation ) *Atomic Physics*
- Feynman (1948) *A space-time approach to non-relativistic quantum mechanics (review article in Revs. Mod. Phys.)*
- Schiff (1955), *Quantum Mechanics*
- French (1958), *Principles of Modern Physics*
- Dicke and Wittke (1960), *Introduction to Quantum Mechanics*
- Merzbacher (1961,1970), *Quantum Mechanics*
- Eisberg (1961,1967), *Fundamentals of Modern Physics*
- Eisle (1964), *Advanced Quantum Mechanics and Particle Physics from an Elementary Approach*
- Feynman and Hibbs (1965), *Quantum Mechanics and Path Integrals*
- French and Taylor (1978), *An Introduction to Quantum Physics*

# What is Laboratory-space?

- Lab space is the 3 dimensional space in which we live. It has 3 visibly-real dimensions, <u>and only 3 dimensions</u>!
- Importantly, the structure of lab space is independent of the system(or systems) under study.
- A volume containing a mixture H atoms, He atoms, $H_2$ molecules, protons, and electrons, still has only 3 dimensions, even though the various species each have different numbers of degrees of freedom.
- Indeed, the existence of particles or systems under study is totally irrelevant to the assumed structure of lab space.
- Greens theorem, Gauss's (divergence) theorem, Stokes theorem, etc. from vector calculus all apply in lab space.
- The lab−space vector, $\mathbf{r}_{Lab} = \hat{\mathbf{e}}_x\, x_{Lab} + \hat{\mathbf{e}}_y\, yLab + \hat{\mathbf{e}}_z\, zLab$ , specifies a unique point in lab space.

# School of thought #1 (à la Einstein, Schrödinger, and Born): The laboratory-space formulation of quantum mechanics

- Following Born, the probability density $\psi_{lab}{}^*(\mathbf{r}_{lab},t)\ \psi_{lab}(\mathbf{r}_{lab},t)$ , when multiplied by the particle's mass or charge gives the local mass or charge density of a beam of non-interacting particles.
- Schrödinger's wave function $\psi_{lab}(\mathbf{r}_{lab},t)$ and Born's associated probability/charge density $\rho_e(\mathbf{r}_{lab},t) = q_e\ \psi_{lab}{}^*(\mathbf{r}_{lab},t)\ \psi_{lab}(\mathbf{r}_{lab},t)$ describe quantities that are to be evaluated at the specific point, $\mathbf{r}_{lab}$ , in lab space. <u>These quantities are, in general, different everywhere in lab space.</u> $\Psi_{lab}(\mathbf{r}_{lab},t)$ is similar to other classical fields, like $\mathbf{E}(\mathbf{r}_{lab},t)$ and $\rho_{mass\text{-}classical}(\mathbf{r}_{lab},t)$.
- The evolution of $\psi_{lab}$ is governed by Schrödinger's (first) equation in lab space, which then depicts "probability" waves propagating in lab space, similarly to classical EM waves, classical sound waves, etc.
- The lab-space formulation allows the definition (via the use of Green's theorem) of Born's "conserved probability current", that "flows" like a fluid in lab space. Born famously used this "current" to predict flux conservation in Rutherford scattering.
- Unfortunately, the lab-space formulation of quantum mechanics works only for single particle systems. <u>It cannot describe N > 1 particle systems</u>.
- Importantly, <u>the lab-space formulation cannot describe entangled-state systems</u>. It is therefore <u>inviable</u>, as a general description of physical processes.

# School of thought #2 (à la Bohr, vonNeumann): The configuration-space formulation of quantum mechanics

School of thought #2 is taught in (pre-1980) standard quantum mechanics textbooks:

- Bethe and Salpeter (1957), *Quantum Mechanics of One- and Two-electron Atoms*
- Bjorken and Drell (1964) *Relativistic Quantum Mechanics;* Bjorken and Drell *(*1965) *Relativistic Quantum Fields*
- Condon and Shortley (1964), *The Theory of Atomic Spectra*
- Dirac (1930,1935, 1947), *The Principles of Quantum Mechanics*
- Landau and Lifshitz (1958, 1965), *Quantum Mechanics*
- Messiah (1961), *Quantum Mechanics* (except that his discussions of the classical limit of quantum mechanics and scattering theory both use lab space)
- Pauling and Wilson (1935), *An Introduction to Quantum Mechanics*
- vonNeumann (1935), *Mathematiche Grundlagen der Quantenmechanik; (*1955, English translation*) Mathematical Foundations of Quantum Mechanics*

# What is Configuration Space?

- Configuration space is an abstract (fictitious) mathematical K−dimensional vector space used for specifying the configuration of a general dynamical system with K "degrees of freedom".
- A mathematical “vector space” is simply one set of number that depend on another set of numbers, and nothing else. (James Kirk does not explore vector space.)
- Configuration space was originally introduced for Hamilton-Jacobi classical mechanics.
- vonNeumann’s book generalizes the configuration space formulation of Quantum Mechanics into the Hilbert space formulation.
- Unlike lab space, the structure of configuration space depends on the system under study.
- The number of degrees of freedom, K, also depends on the system under study.
- A particle’s “degrees of freedom" are not "diminsions of space-time”!

# School of thought #2: (à la Bohr and vonNeumann): The configuration-space formulation of quantum mechanics

- The wave function $\psi_{config}(q_{1,config}, q_{2,config}, \ldots, q_{K,config}, t)$ describes a quantity that depends on the system's K "degrees of freedom": $q_{1,config}, q_{2,config}, \ldots, q_{K,config}$.
- Schrödinger's equation (in school #2) now describes the evolution of $\psi_{config}$ in an abstract mathematical K−dimensional vector space.
- For a system comprised of say, N = 2 mobile particles, each with spin, the wave function then has the form $\psi_{config}(\mathbf{r}_{1,config}, s_{1,config}, \mathbf{r}_{2,config}, s_{2,config}, t)$, where $\mathbf{r}_{1,config}$ (= $x_{1,config}$, $y_{1,config}$ $z_{1,config}$) and $s_{1,config}$, are the position and spin of particle 1, and $\mathbf{r}_{2,config}$ and $s_{2,config}$ are the position and spin of particle 2. It has K = 8 degrees of freedom.
- <u>Importantly</u>, the wave function $\psi_{config}(q_{1,config}, q_{2,config}, \ldots, q_{K,config}, t)$ has no lab-space dependence. Unlike $\psi_{lab}(\mathbf{r}_{lab},t)$, there is no specific point, $\mathbf{r}_{lab}$, in lab space where it is to be evaluated. ($\mathbf{r}_{lab}$ would be a disallowed "hidden variable" if $\psi_{config}$ were to depend upon it, and transformation theory would not work.)
- That means that $\psi_{config}$ and $\psi_{config}^* \psi_{config}$ are independent of $\mathbf{r}_{lab}$ ! → They have the same values everywhere in lab space! → There are no "waves of probability" propagating in lab space, unlike in the lab-space school of thought!

# Born's Ambiguity

- Born's textbook (erroneously) claims that the two wave functions $\psi_{lab}$ and $\psi_{config}$ , and their associated argument spaces ($\mathbf{r}_{lab}$ ,t) and ($q_{1,config}$ , $q_{2,config}$ , … , $q_{K,config}$ , t) are equivalent, because $\psi_{lab}^* \psi_{lab}$ and $\psi_{config}^* \psi_{config}$ both yield probability densities.
- <u>They are not equivalent!</u> A particle's degrees of freedom are <u>not</u> dimensions in lab space. Their values do not specify the value of $\psi_{config}$ at the position $\mathbf{r}_{lab}$. Their argument spaces are not equivalent.
- Correspondingly, there are no associated "probability" waves and probability currents propagating in lab space for N > 1 entangled particle systems! (Dicke and Wittke, and Merzbacher both express alarm over this fact and the associated loss of a conceptual model.)
- The N-particle Schrödinger's equation (in configuration space) in the special case, N=1, <u>does not reduce to the single-particle Schrödinger's equation (in lab space).</u>

# The N-particle Schrödinger's equation (in configuration space), in the N = 1 special case, should reduce to the single-particle Schrödinger's equation (in lab space). It does not!

- Configuration space is necessary for providing a quantum mechanical description of $N > 1$ particle systems. It provides entanglement!
- In any rational self-consistent formulation of quantum mechanics, simply changing the number of particles from $N = 1$ to $N > 1$ in the system under study should not require changing the space in which Quantum Mechanics is formulated.
- For self-consistency, configuration space is necessary for providing a quantum mechanical description of an $N \geq 1$ particle systems. (i.e. for <u>both</u> $N = 1$ and $N > 1$).
- <u>The lab-space formulation of Quantum Mechanics is a form of Local Realism</u>. Correspondingly, the single-particle Schrödinger's equation formulated in lab space is disallowed by experiment. (Recall, it's dead, Jim.)
- Therefore, to be self consistent, the configuration-space formulation should <u>always</u> be used, even for a single particle system. <u>Sadly, the single-particle Schrödinger's equation formulated in lab space should never be used, even if it help to make Quantum Mechanics easier to understand! There are no probability waves!</u>

# The use of configuration space also has technical difficulties

- Greens theorem, Gauss's theorem, Stokes theorem, etc. do not apply in configuration space. They describe the geometrical properties of 3D lab space (only).
  - Born's conserved currents cannot be constructed for N > 1 entangled particle systems, because Green's Theorem does not apply in configuration space.
  - Born, Landau and Lifschitz and Messiah erroneously ignore this fact.
  - Nobody ever explains what the flow of a current in configuration space actually means.
- Time is inherently a lab-space concept! vonNeumann (1955, p354) points out an "unhappy", "essential weakness" of the configuration-space formulation of Quantum Mechanics. That is its inherently non-relativistic character.
  - For N > 1 particle systems, Lorentz transformations are not possible in configuration space, because time is distinguished from and independent of ($x_{j,config}$, $y_{j,config}$, $z_{j,config}$) for the j'th particle's position.
  - That is, there is no associated $t_{j,config}$, which is needed because time advances differently for each high-speed particle.
- Bethe and Salpeter encounter related difficulties for the calculation of relativistic corrections to the spectrum of Helium via the Breit equation, esp. for high-Z Helium-like ions (The Breit equation is the Dirac equation equivalent for 2 electron atoms).

# Some important conceptual difficulties associated with the use of configuration space

- Unfortunately, Schrödinger's equation in configuration space describes the evolution of $\psi_{config}$ in an abstract mathematical K−dimensional vector space, only!
- The conceptual models of probability waves propagating in (lab) space and Born's conserved probability current flowing in lab space are untenable, in general!
- Configuration-space wave functions, propagating in configuration space, and the flow of a current in configuration space are difficult to visualize (impossible for me, for Dicke and Wittke, for Merzbacher, and evidently for Einstein also) while visualizing the propagation of waves in lab space was straight-forward. (Because, lab-space propagation is dead, Jim!)

# Bohr and Einstein were talking past each other – (as a consequence of Born’s ambiguity)

- Bohr and Einstein both evidently bought Born’s ambiguity.
- Bohr was assuming that Quantum Mechanics is formulated in configuration space.
- Einstein was assuming that Quantum Mechanics is formulated in lab space.
- Since configuration space is a totally artificial abstract mathematical space, there is no prohibition for superluminal wave propagation in configuration space (whatever that means). Einstein’s problems then disappear for Bohr.
- EPR’s locality (action-at-a-distance) arguments carry no weight in configuration space. Einstein’s problems then disappear for Bohr.
- vonNeumann’s wave-function collapse occurs in only configuration space, but not in lab space. It also does not require superluminal communication. The measurement problem disappears. Einstein’s problems then correspondingly disappear for vonNeumann.

# Entanglement has a long range in both time and space

- Entanglement in space has been experimentally demonstrated (via CHSH inequality violation) to persist for over a thousand kilometer particle separation – e.g. by the Chinese Micius quantum-encrypted communications satellite. There appears to be no limit to the range.
- Messiah’s quantum mechanics textbook shows that if two particles are independent and do not interact, and have never interacted in the past, then they are not entangled and remain not entangled in the future, unless they interact in the future.
- The converse is also true. If they ever interacted in the past, then they are entangled. Under Schrödinger (unitary) evolution, They remain entangled, even after if they cease to interact.
- Spitzer points out that the classical Coulomb interaction between any pair of particles has infinite range and the associated scattering cross-section is infinite. (The differential scattering cross-section diverges at small angles.) → All charged widely-separated particles interact, at least very weakly.
- Thus, since virtually all charge particles have been interacting (even weakly) since the early epochs of the universe, all charged particles (e.g. protons and electrons) are at least a little bit entangled.
- Under Schrödinger equation evolution, wave functions grow exponentially in time.
- Being a little bit entangled is thus like being a little bit pregnant. The importance of both conditions depends on how long you wait.

# Final Quandary

- It is noted above that Einstein’s General Relativity is a deterministic form of Local Realism.
- But Local Realism is dead (Jim).
- General Relativity requires a lab-space formulation.
- Quantum Mechanics requires an explicitly unambiguous configuration-space formulation.
- Any quantum mechanical extension of General Relativity correspondingly must be formulated in configuration space.
- Is a quantum-mechanical configuration space extension of General Relativity possible?
- Entanglement is not allowed in lab space.
- Black holes are formed in General Relativity by gravity’s distortion of lab space.
- → Can a black hole’s “spin” be quantized?
- → Can a black hole pair be in a definite spin state, whereupon the individual black-hole spins are entangled?
- → Will spin-component measurements of the individual black holes violate a Bell inequality?