From Bell inequalities to Quantum Measurements

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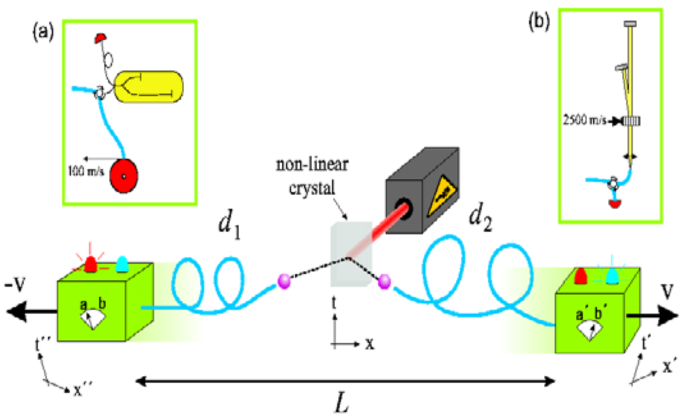

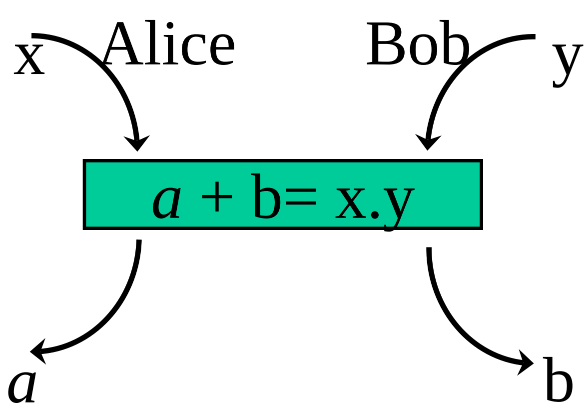

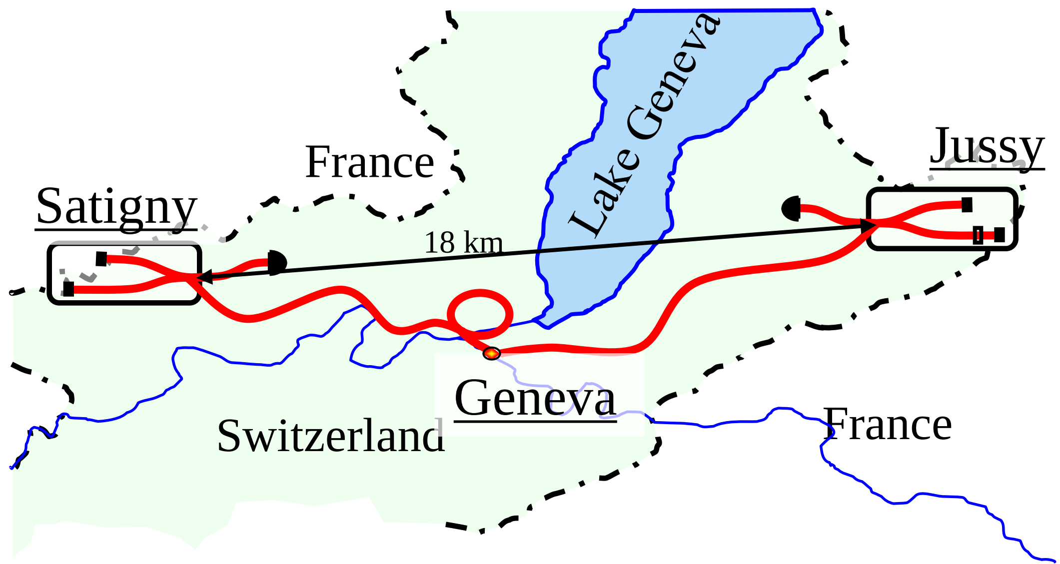

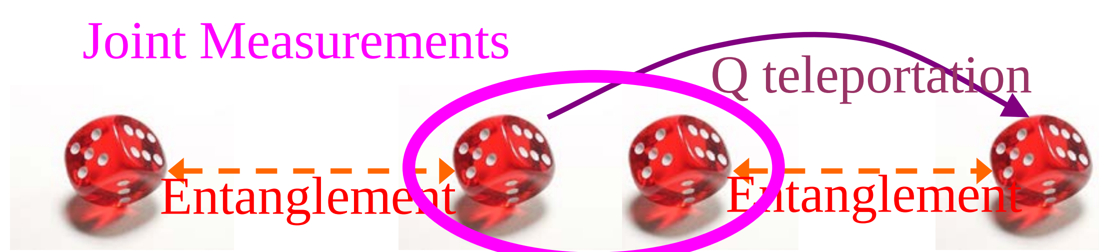

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In the 1970's there were two main challenges affecting the foundations of quantum physics:

1. Quantum Non-Locality, i.e. violation of Bell inequality
2. The Quantum Measurement Problem, i.e. the Schrödinger eq. is interrupted whenever the system encounters a bunch of atoms with a sticker saying "measurement apparatus".

General consensus :

Quantum non-locality: kind of solved,

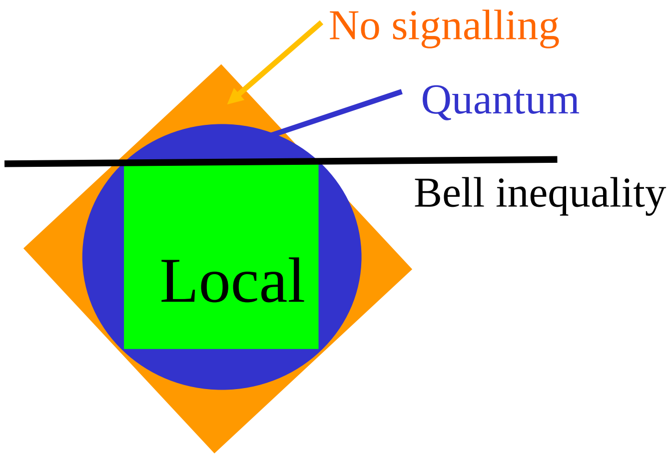
certainly lots of progress,

Quantum measurement problem: not much progress,

certainly no agreement.

Bell inequalities, lots of progress

- Theoretical physics:
  communication cost to simulate quantum correlations
- Conceptual physics:
  PR-box (conceptual tool)
- Mathematical physics:
  local polytope
  $\Rightarrow$ many Bell inequalities



- Experiments

Experiments

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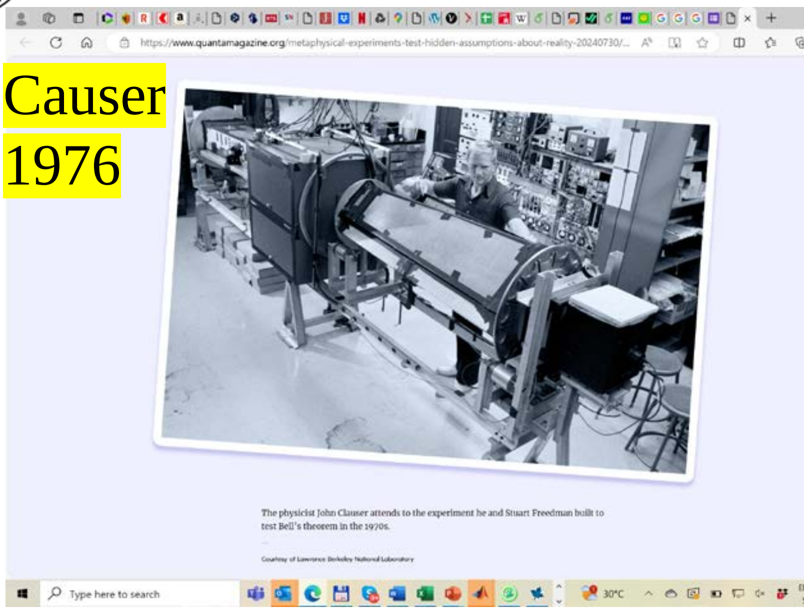
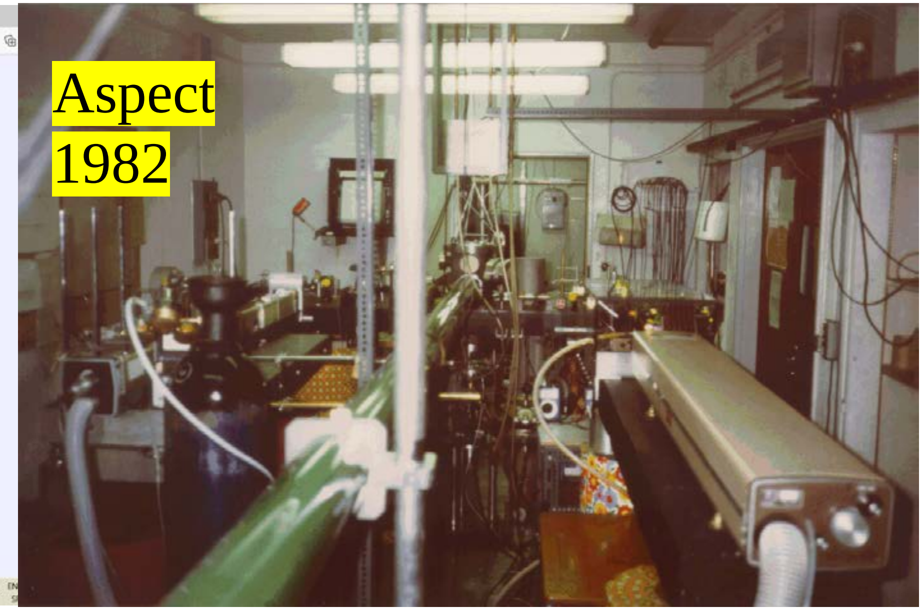
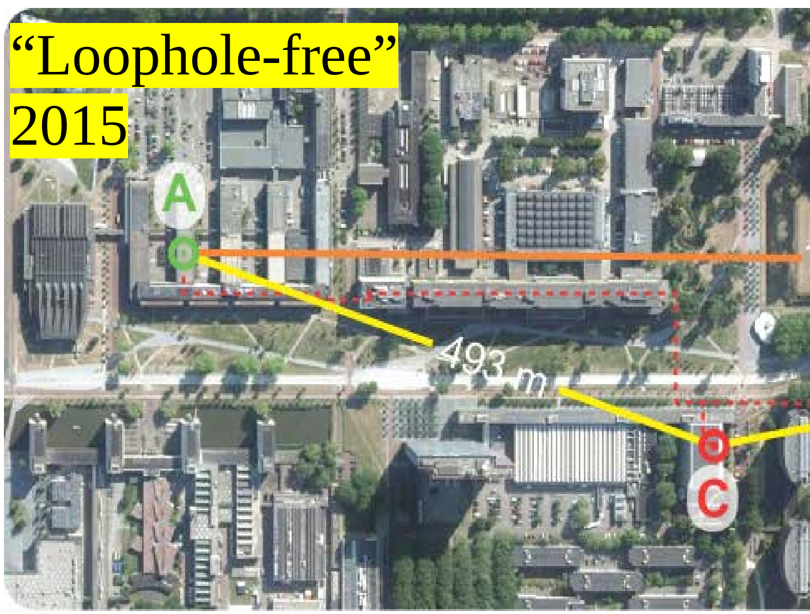
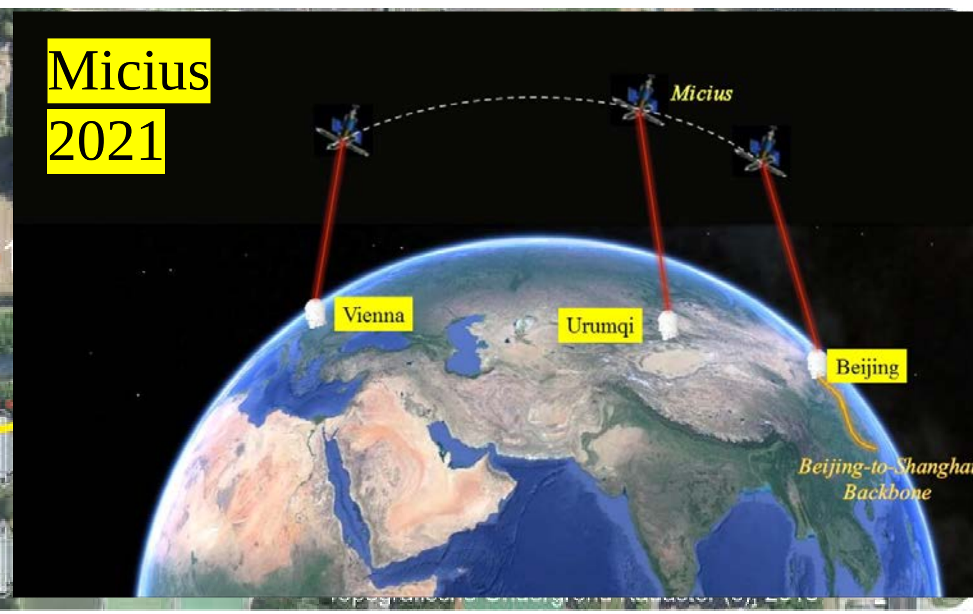




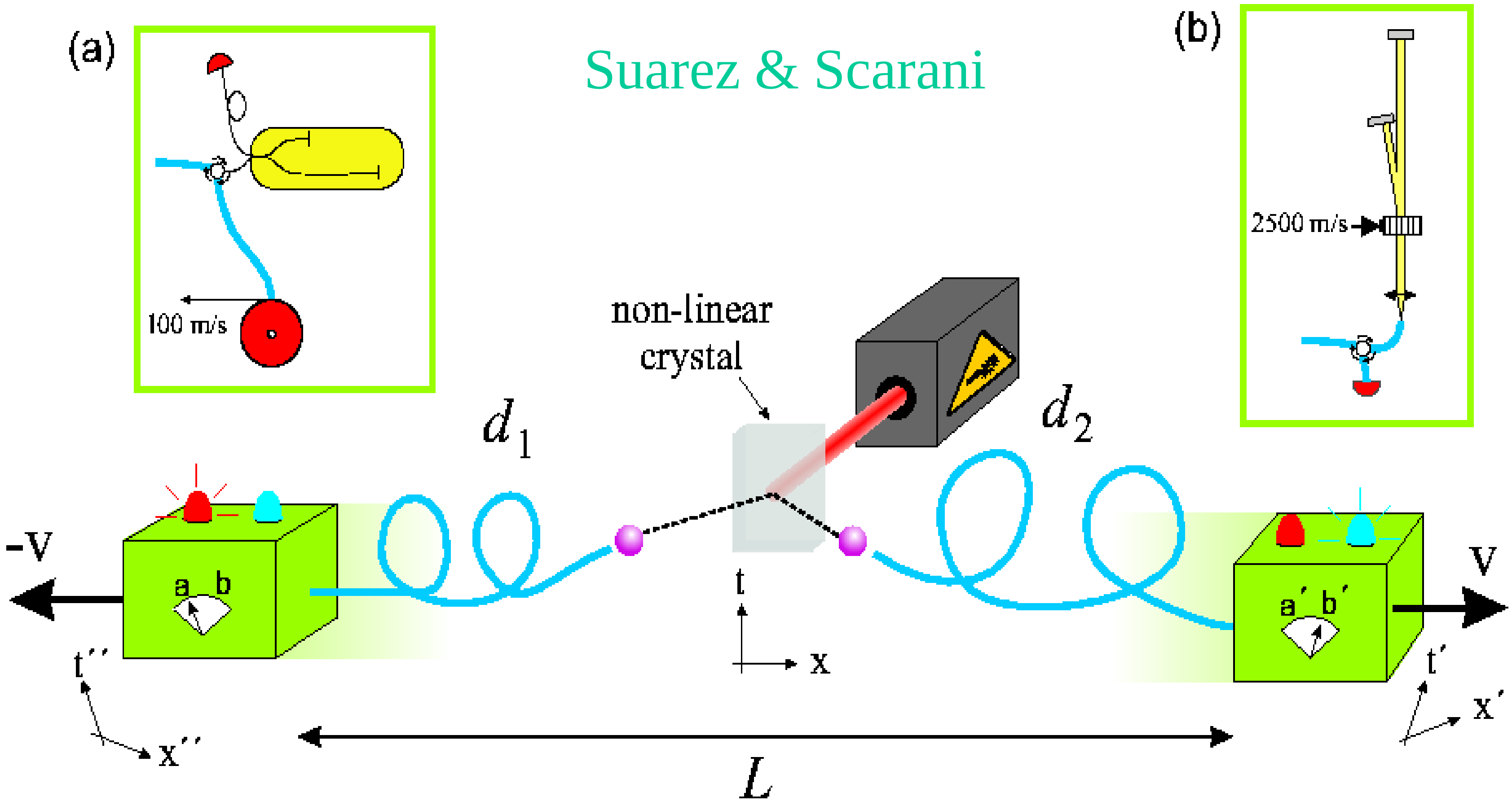




Further experiments: before-before configurations

NG, *Sundays in a quantum engineer's life*, quant-ph/0104140

in *Quantum [Un]speakable*, pp 199-208, ed. R.A. Bertlmann and A. Zeilinger, Springer 2002



PRL 88,120404,2002; J.Phys.A 34,7103,2001; Phys.Lett.A 276,1,2000

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There is no *c* in $\mathcal{H}$

Alice | Bob

Let's test whether something is going on behind the scene, as John Bell used to say.
Let's test not only common causes, but also direct causation.

All experiments so far find $v \geq$ 50 to 100 thousand times c

Which goal should the community set?

c/speed sound $\approx 10^6$

next generation exp should aim at $v \approx 10^7$ c

2 orders improvement is feasible:
x10 thanks to snspd
x10 thanks to longer distances
+ faster data acquisition

Salart et al., Nature 454, 861, 2008
Cocciaro et al., PLA 375, 379, 2011
J-W Pan's group, PRL 110, 260407, 2013
L. Santamaria et al., scientific report 13.8201 (2023)

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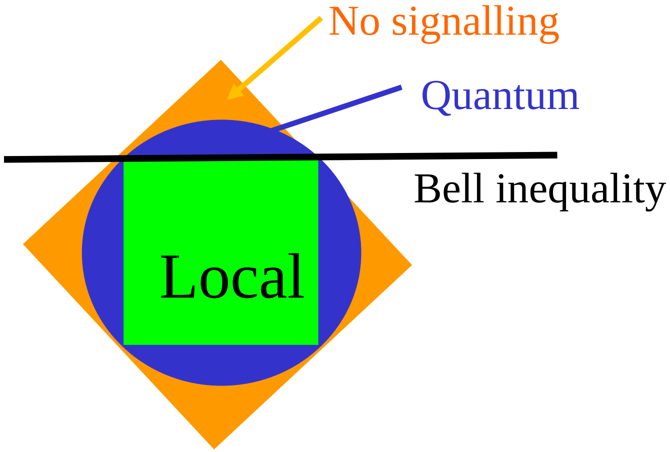
Bell inequalities

- Theoretic physics:
  communication cost to simulate quantum correlations
- Conceptual physics:
  PR-box (conceptual tool)
- Mathematical physics:
  local polytope ⇒ many Bell ineq

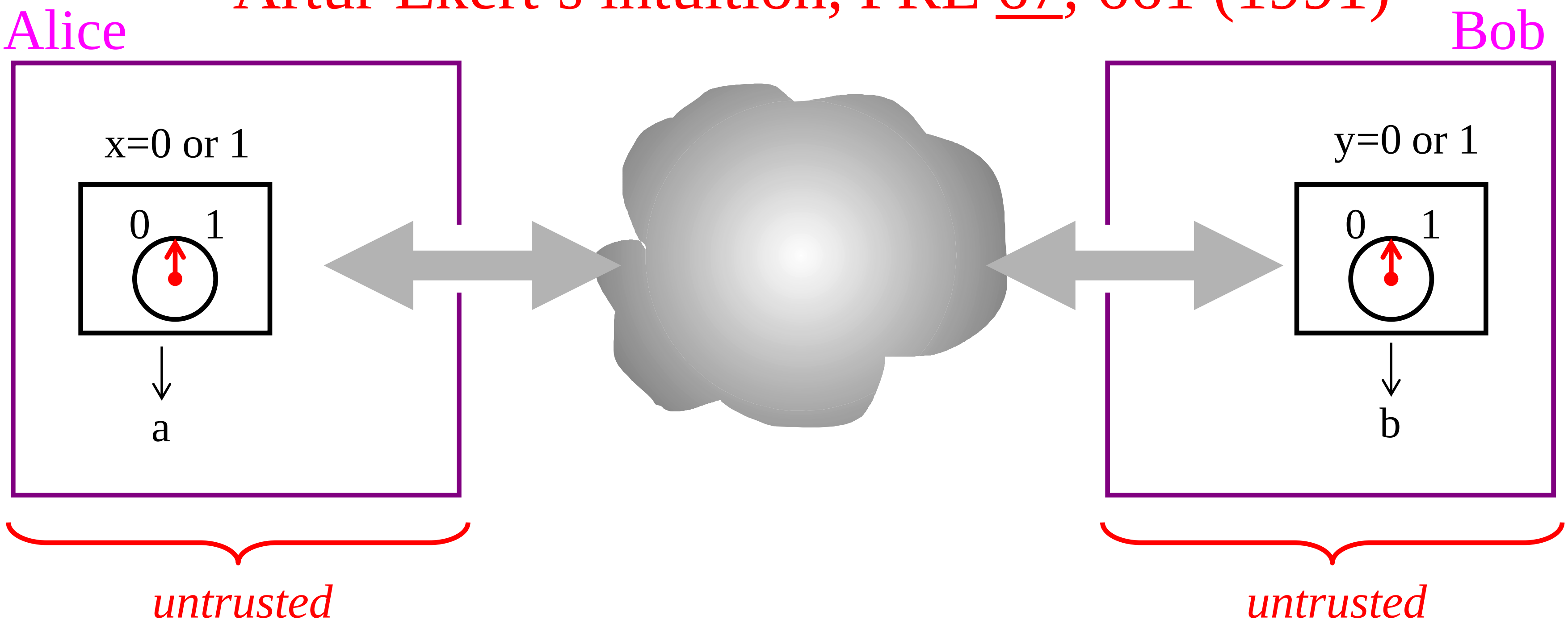


- Experiments

- Applications

Exploit quantum nonlocality for cryptography

Artur Ekert's intuition, PRL 67, 661 (1991)



If p(a,b|x,y) violates some Bell inequality, then p(a,b|x,y) contains secrecy irrespective of any detail of the implementation !

Proofs: PRL 97, 120405, 2006; PRL 98, 230501, 2007; NJP 11, 1, 2009

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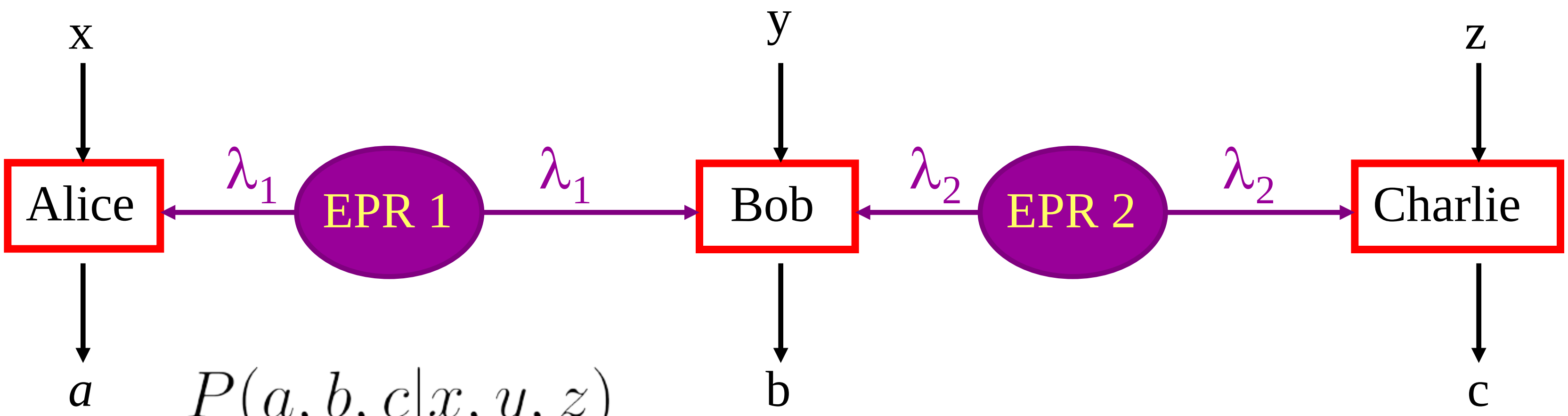
Network non-locality

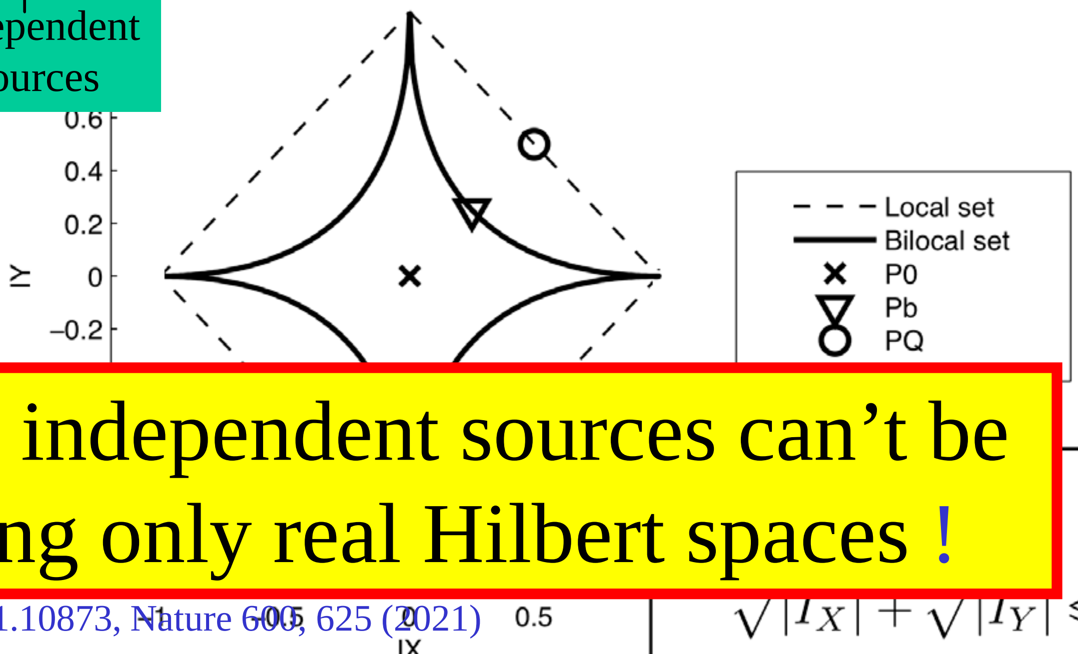
Branciard, NG, Pironio
PRL 104, 170401-1/4, 2010



$$P(a,b,c|x,y,z) = \sum_{\lambda_1,\lambda_2} p(\lambda_1)p(\lambda_2)p(a|x,\lambda_1)p(b|y,\lambda_1,\lambda_2)p(c|z,\lambda_2)$$

independent sources

The set of bilocal $p_{biloc}(a,b,c|x,y,z)$ is **not convex**:



$\sqrt{|I_X|} + \sqrt{|I_Y|} \leqslant 1$

Networks with independent sources can't be described using only real Hilbert spaces !

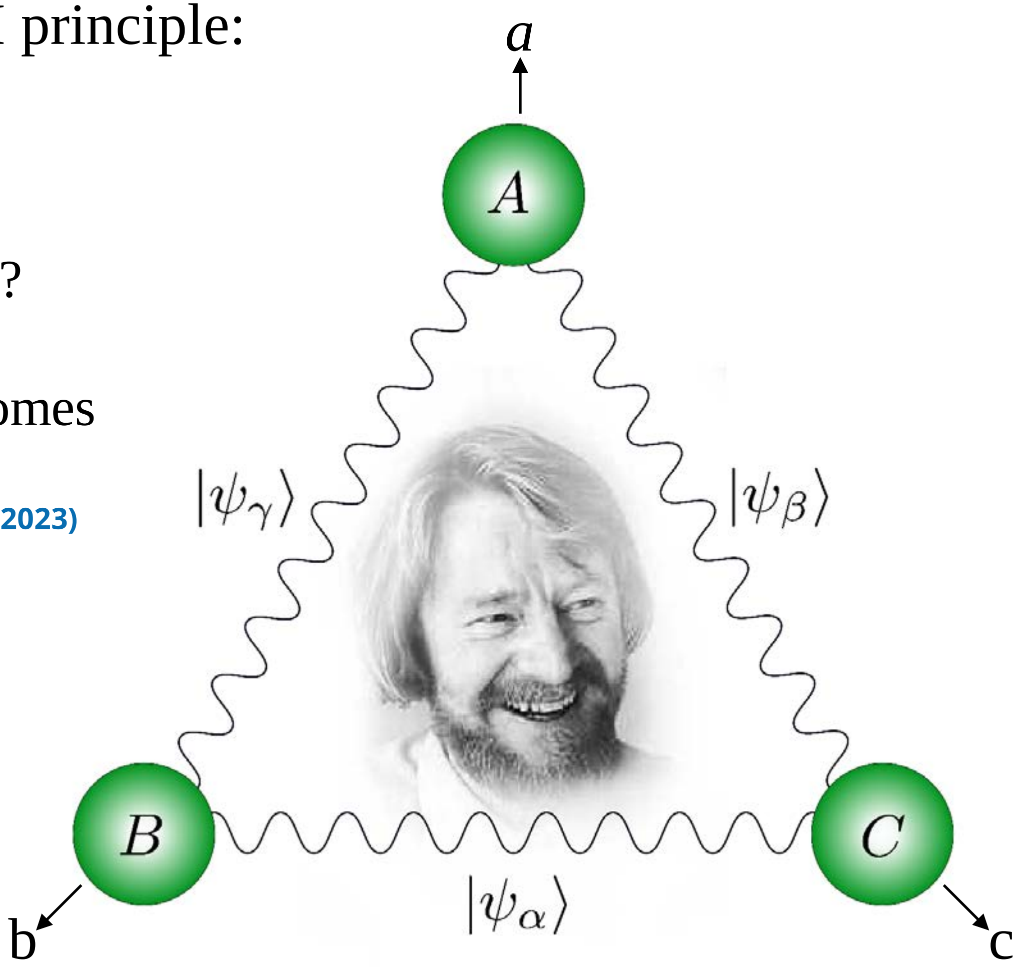
Review: M.Renou et al., arXiv:2101.10873, Nature 600, 625 (2021)
Tavakoli et al., Rep. Prog. Phys. 85, 056001 (2022)

Triangle without inputs

P(a,b,c) satisfies the NSI principle:
- No-Signalling
- Independent sources

1. How to prove quantumness? Where is $\hbar$?
2. How to prove that the outcomes contain some randomness?

Sekatski, Boreiri, Brunner, PRL 131, 100201 (2023)
Boreiri et al., PRL 134, 010202 (2025)



Review on Network Non-Locality:
Tavakoli et al., Rep. Prog. Phys. 85, 056001 (2022)

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How does Nature perform the trick ?

- How can these two locations out there in space-time know about each other ?
- How does an event A know that it is nonlocally correlated to another event B ?
- Who keeps track of who is entangled with whom ?

Quantum nonlocality : how does nature do it ?, Science, 326, 1357, 2009;
arXiv:0912.1475

Conclusion from non-locality

⇒ There is no spooky action at a distance: there is not a first event that influences a second event.

⇒ Quantum correlation just happen, without any time-ordering, somehow from outside space-time !

(there is no story in space-time that tells us how it happens)

⇒ **there is no mechanical explanations of quantum correlations**

⇒ Materialism is false !

(nature is not a sort of legoland)

The deep Q Measurement Problem

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- **Physics is all about extracting information about**

  How Nature Does it

- **Extracting information = performing measurements.**

  Measurements have outcomes

- **Hence the Quantum Measurement Problem is a serious physics problem:**

  Without a resolution, Q theory is not physics.

- **A resolution will lead to new physics.**
- There is no reason to give up realism, nor to believe in determinism.
- realism ≠ determinism.

NG, *Non-realism: Deep Thought or a Soft Option?*, Found. Phys. 42, 80-85 (2012); arXiv:1602.01497 & 1701.08300
Flavio Del Santo, *Striving for Realism, not for Determinism: Historical Misconceptions on Einstein and Bohm*, APS News}, 18(5)

Instead of fighting the deep measurement problem heads on, let's tame it, let's get more familiar with quantum measurements

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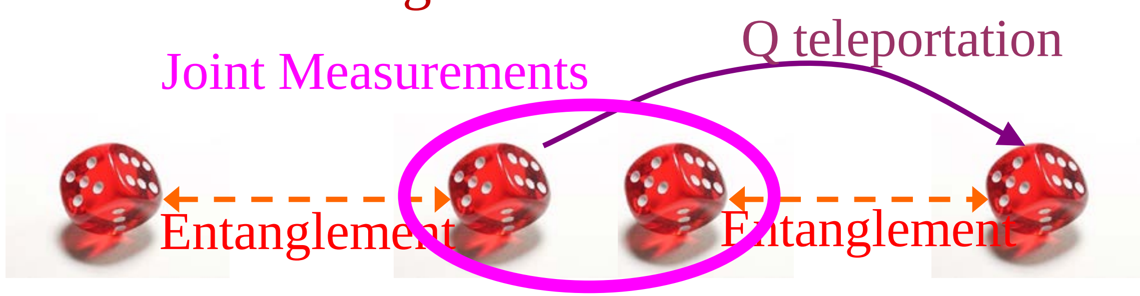
Today we know a lot about
- POVMs
- Weak measurements
- Conditions under which measurements are compatible or not
- etc

But what do we know about joint measurements, measurements whose eigenstates are entangled?



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Today we know a lot about
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But what do we know about joint measurements, measurements whose eigenstates are entangled?

Most physicists – even within the quantum information community – would not be able to mention any joint measurement beyond the famous Bell State Measurement.

Impossible Measurements on Quantum Fields

RAFAEL D. SORKIN

arXiv:gr-qc/9302018

Department of Physics, Syracuse University, Syracuse NY 13244-1130

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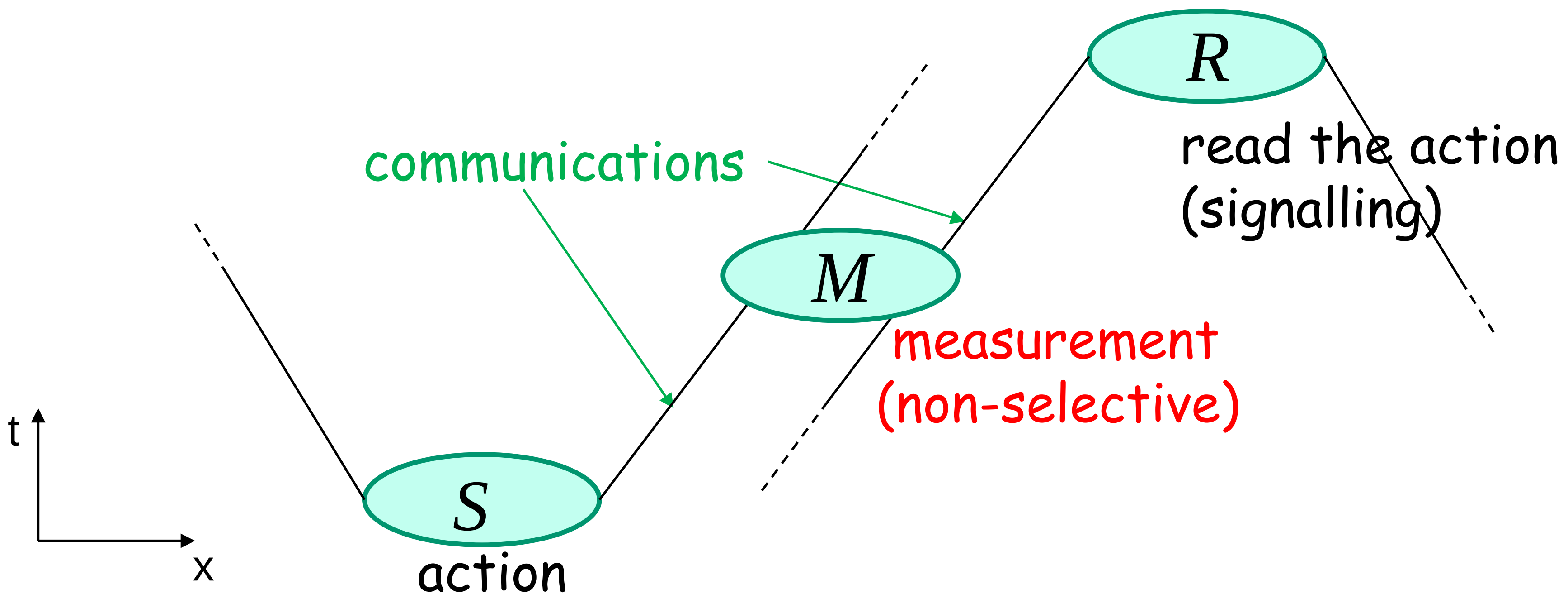
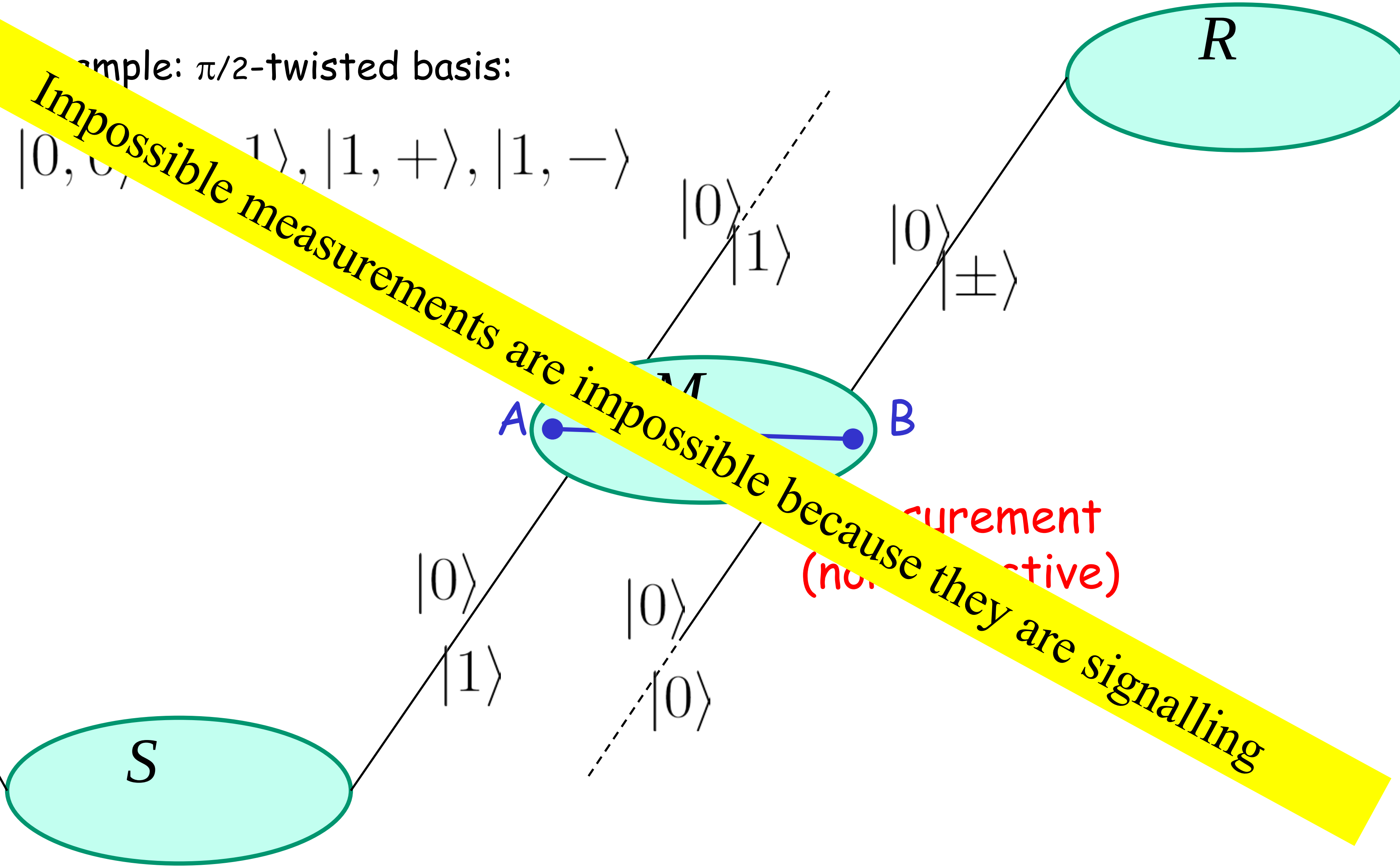


- The measurement covers some space-like region. Hence, impossible measurements are measurements of NL (non-local) variables.

Impossible Measurements

…mple: $\pi/2$-twisted basis:

$|0, 0\rangle$, … $1\rangle, |1, +\rangle, |1, -\rangle$



Impossible measurements are impossible because they are signalling

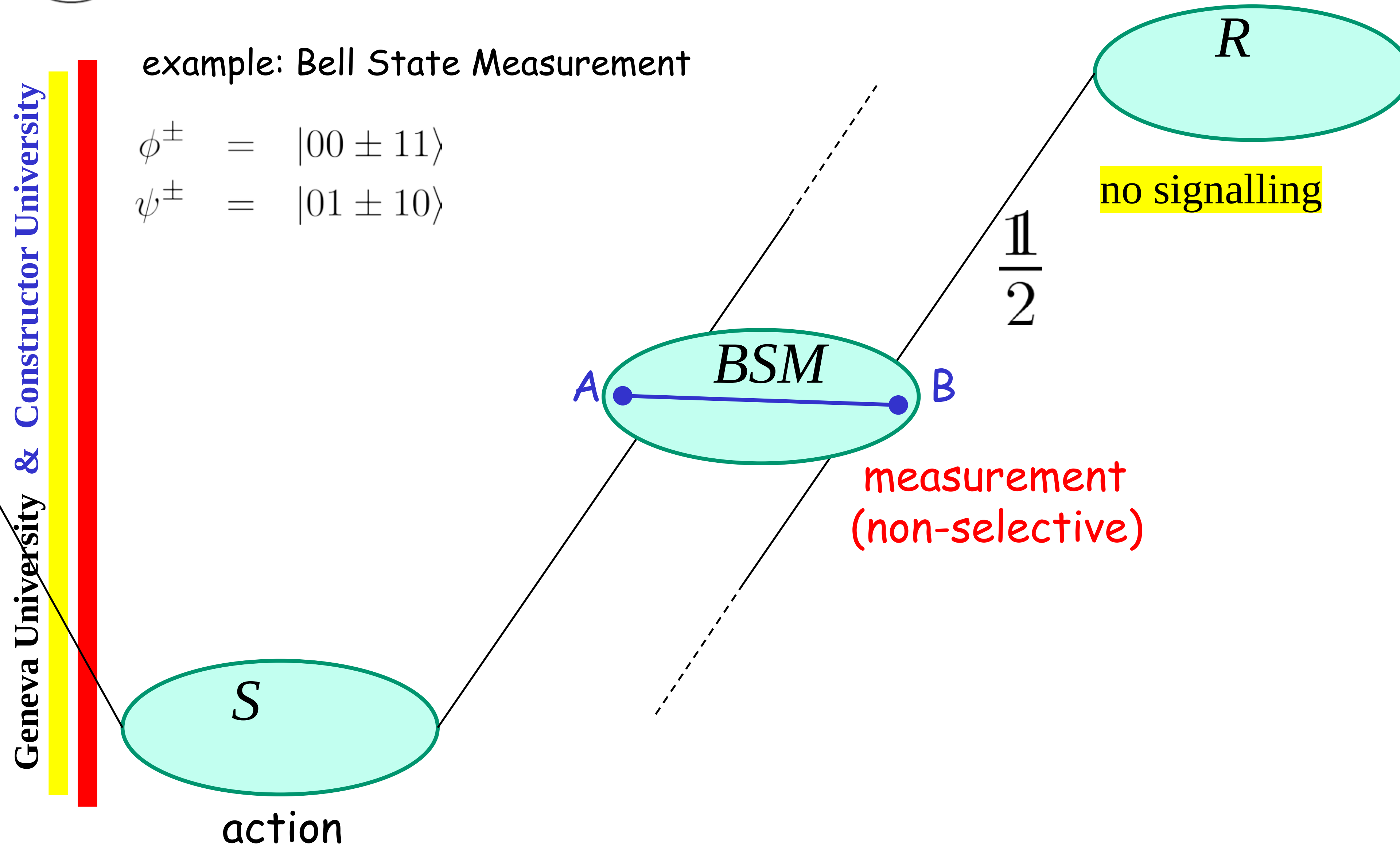
action σ_x

Flavio del Santo & NG, arXiv:2311.13644 Quantum 8, 1267 (2024)

Impossible Measurements

example: Bell State Measurement

$$\phi^\pm = |00 \pm 11\rangle$$
$$\psi^\pm = |01 \pm 10\rangle$$



Flavio del Santo & NG, arXiv:2311.13644 Quantum 8, 1267 (2024)

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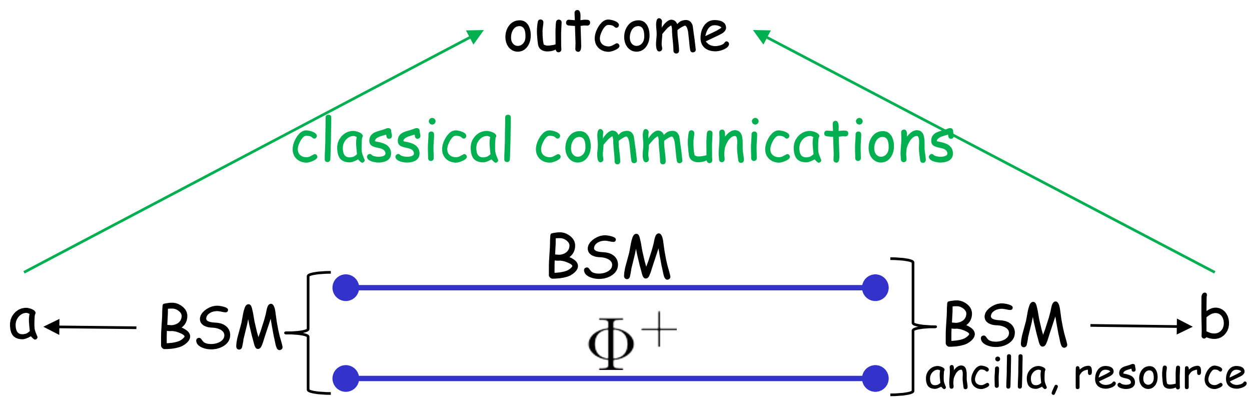
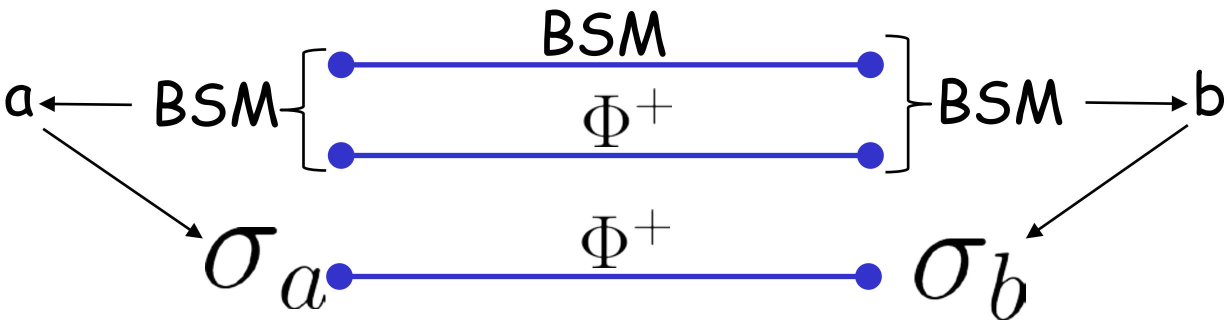
BSM: Bell State Measurement



The outcomes are s.t. the number of Φ is even and the number of + is also even.

⇒ recovers the Born rule,
is not signalling,
is localizable (all quantum part is done locally),
but is not ideal (not immediately reproducible, not projective)

Ideal BSM



Now, the BSM is localized and ideal (up to some swaps).

<u>Theorem</u>: (Popescu-Vaidman)
The BSM is the <u>only</u> joint NL measurement of 2 qubits that can be measured ideally without signalling.

Note: the BSM is not a typical measurement, but is exceptional !

Flavio del Santo & NG, Quantum 8, 1267 (2014)
Jef Pauwels, Alejandro Pozas Kerstjens, Flavio del Santo, NG, PRX 15, 021013 (2025)

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Localizable Measurements

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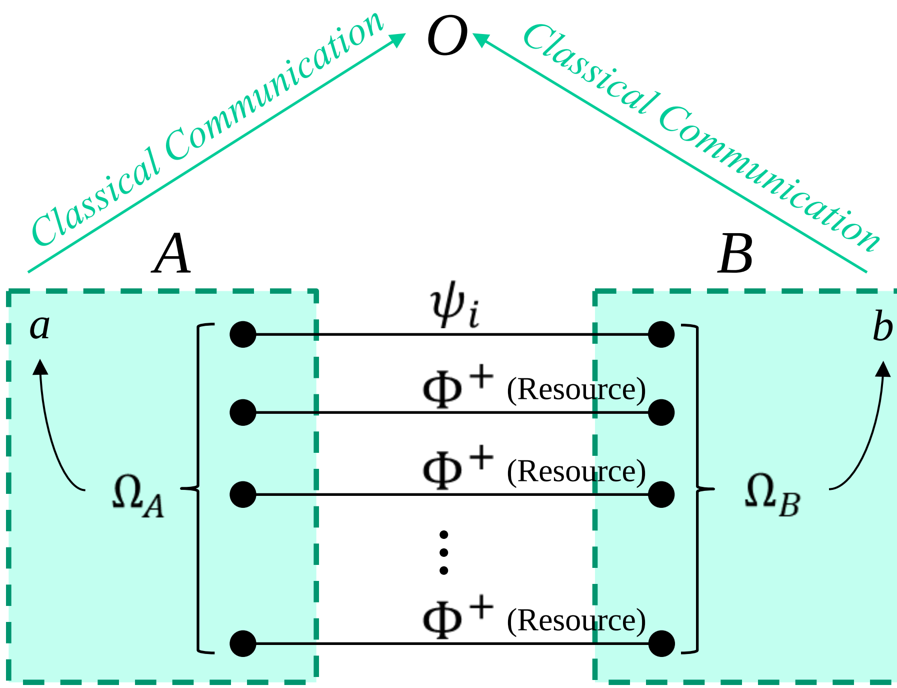

Theorem: All measurements, any dimension, any nb of parties, are localizable (with infinite # ebits).

(Groisman-Reznik, Vaidman)

In a nutshell: "impossible" measurements are localizable, hence possible, but can't always be ideal.

Flavio del Santo & NG, Quantum 8, 1267 (2014)
Jef Pauwels, Alejandro Pozas Kerstjens, Flavio del Santo, NG, PRX 15, 021013 (2025)

Classification of Joint Measurements

Let's classify joint measurements in function of the entanglement cost necessary to localize them.

Obviously, with 0 entanglement only product measurements can be localized

$\Rightarrow$ the set of all joint quantum measurements has some structure, from the simplest to the most complex.

The structure is somewhat the analogue for measurements of the local polytope for quantum non-locality.

This structure is related to the Clifford hierarchy.

Flavio del Santo & NG, Quantum 8, 1267 (2014)
Jef Pauwels, Alejandro Pozas Kerstjens, Flavio del Santo, NG, PRX 15, 021013 (2025)

Localizable NLM with 1 ebit (2^{nd} level)

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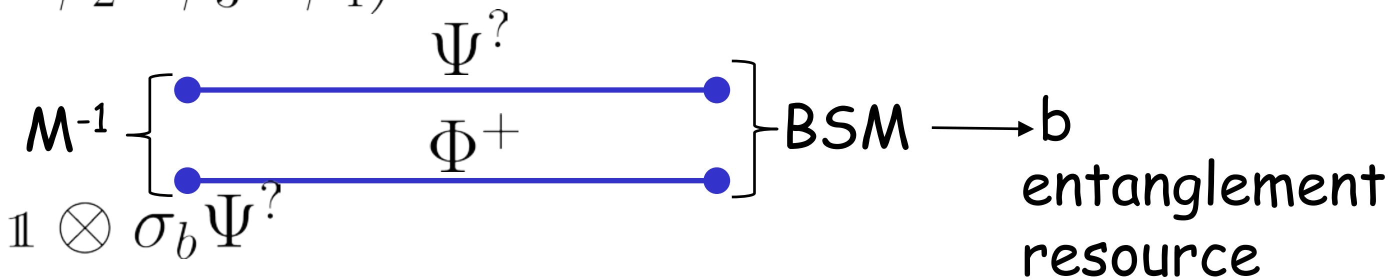
$$M = (\psi_1 \;\; \psi_2 \;\; \psi_3 \;\; \psi_4)$$



$$M^\dagger \cdot \mathbb{1} \otimes \sigma_b \cdot M = \tilde{P}_b \Phi_b \equiv P_b$$

= permutation + phases of 00,01,10,11

Such NLM are not ideal: they are not immediately reproducible

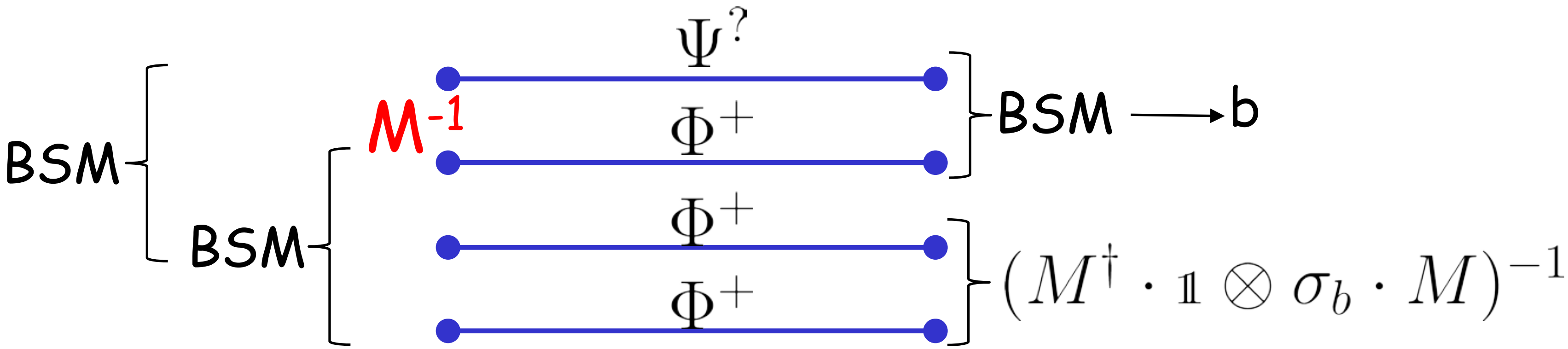
Theorem (2^{nd} level of the hierarchy)

The only measurements localizable with 1 ebit, but not with 0 ebits are the

1. $\pi/2$-twisted basis measurement, and the
2. Bell State Measurement.

Flavio del Santo & NG, Quantum 8, 1267 (2014)
Jef Pauwels, Alejandro Pozas Kerstjens, Flavio del Santo, NG, PRX 15, 021013 (2025)

Localizable NLM with 3 ebit (3rd level)



$$(M^\dagger \cdot \mathbb{1} \otimes \sigma_b \cdot M) \cdot \sigma_{a_1} \otimes \sigma_{a_2} \cdot (M^\dagger \cdot \mathbb{1} \otimes \sigma_b \cdot M)$$

= permutation+phases of 00,01,10,11

<u>Theorem (3rd level of the hierarchy)</u>

The only measurements localizable with 3 e-bit, but not with 1 e-bits are the:

1. partial BSM $\{\psi^\pm, |00\rangle, |11\rangle\}$
2. elegant joint measurement (EJM), see next slide,
3. another member of the EJM family
4. two members of the BS family

arXiv:2307:06998
PR Research 6,023085(2024)

Jef Pauwels, Alejandro Pozas Kerstjens, Flavio del Santo, NG, PRX 15, 021013 (2025)

The Elegant Joint Measurement (EJM)

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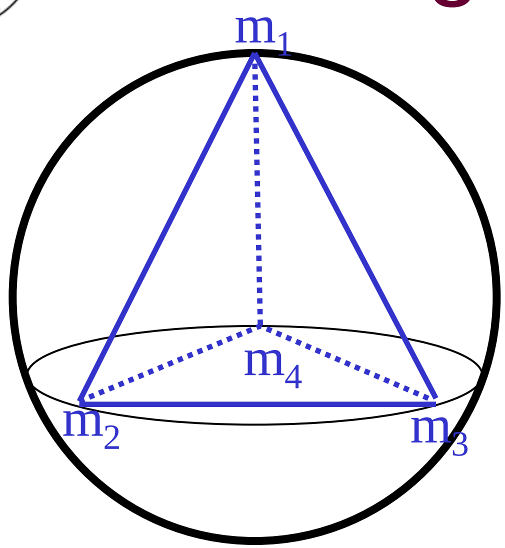

Look for 4 partially entangled and mutually orthogonal states with same degrees of entanglement and with partial states along the vertices of the tetrahedron.

$$\vec{m}_j = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

$$\left|\Phi_j\right\rangle = c_0\left|\vec{m}_j, -\vec{m}_j\right\rangle + q_0\left|-\vec{m}_j, \vec{m}_j\right\rangle$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}\left|\vec{m}_j, -\vec{m}_j\right\rangle + \frac{\sqrt{3}-1}{2\sqrt{2}}\left|-\vec{m}_j, \vec{m}_j\right\rangle$$

$\left\langle\Phi_j\middle|\Phi_i\right\rangle = \delta_{ji}$ & c_0, c_1 are real

$$Tr_B\left(\left|\Phi_j\right\rangle\left\langle\Phi_j\right|\right) = \frac{1}{2}\left(1 + \frac{\sqrt{3}}{2}\vec{m}_j\right) \qquad Tr_A\left(\left|\Phi_j\right\rangle\left\langle\Phi_j\right|\right) = \frac{1}{2}\left(1 - \frac{\sqrt{3}}{2}\vec{m}_j\right)$$

iso-entangled with tangle = ¼. Localizable, but not ideal meas.

NG, Entropy, «25 years of Q teleportation», 21,325 (2019)

Generalized EJM

J. Pauwels, NG
arXiv:250901852
J. Pauwels et al.
arXiv:2509.011851

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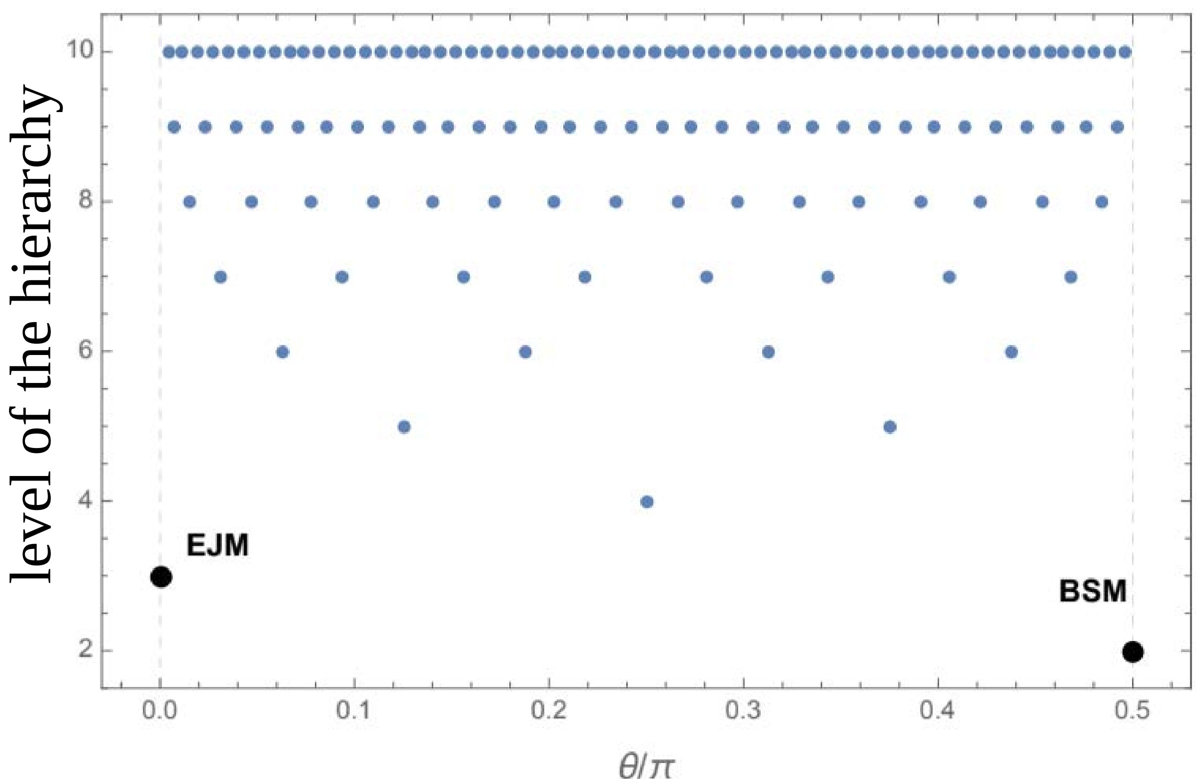
$$|\Phi_j\rangle = \frac{\sqrt{3}+e^{i\theta}}{2\sqrt{2}}|\vec{m}_j, -\vec{m}_j\rangle + \frac{\sqrt{3}-e^{i\theta}}{2\sqrt{2}}|-\vec{m}_j, \vec{m}_j\rangle$$



The Multiqubit Elegant Joint Measurement

J. Pauwels, NG
arXiv:250901852
J. Pauwels et al.
arXiv:2509.011851

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$$|\psi\rangle = \sum_{\boldsymbol{s}\in\{\pm\}^n} c_{\boldsymbol{s}} \, \big| s_1 \textcircled{Δ} \otimes s_2 \textcircled{Δ} \otimes \cdots \otimes s_n \textcircled{Δ} \big\rangle,$$

$$G^{(n)}_{\text{tet}} \equiv \langle Z_i Z_{i+1},\, X^{\otimes n} \rangle \qquad \mathcal{B} = \left\{ g\,|\psi\rangle \,\middle|\, g \in G^{(n)}_{\text{tet}} \right\}$$

3-qubit EJM:

$$\begin{aligned} |\psi\rangle = c_0 & \,|-\vec{m}_1, -\vec{m}_1, -\vec{m}_1\rangle \\ & + c_1 \left(|\vec{m}_1, -\vec{m}_1, -\vec{m}_1\rangle + \text{perms.} \right) \\ & + c_2 \left(|\vec{m}_1, \vec{m}_1, -\vec{m}_1\rangle + \text{perms.} \right) \\ & + c_3 \,|\vec{m}_1, \vec{m}_1, \vec{m}_1\rangle \ . \end{aligned}$$

Higher dimensions:
J. Czartowski and K. Zyczkowski, Quantum 5, 442 (2021)

Relation to Clifford hierarchy

J. Pauwels, NG
arXiv:250901852
J. Pauwels et al.
arXiv:2509.011851

Recursive algebraic hierarchy organizing all unitaries by their action on Paulis

$$C_k = \{U \in U(2^n) | UPU^\dagger \in C_{k-1} \ \forall P \in P_n\}, \quad C_1 = P_n$$

Can be understood as **complexity of nonlocal computation**

Similarly, localizability is an algebraic hierarchy:

$$V_1 \subsetneq V_2 \subsetneq \dots \subsetneq V_k \subsetneq \dots$$

$$C_k \subsetneq V_k$$

Intuition: Measurements are a subset of all computations

What does that teach us about the deep quantum measurement problem?

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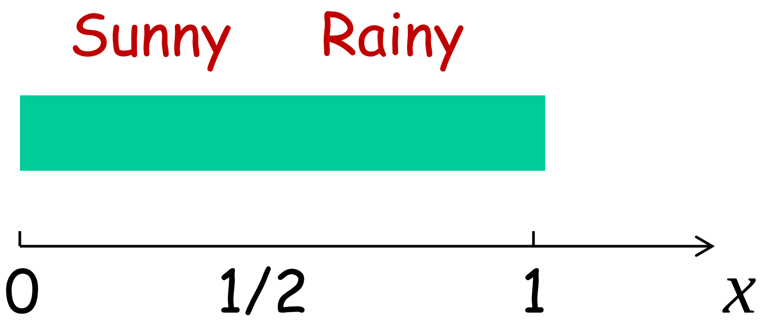
May be as much as
- local polytopes,
- communication cost of simulating non-local correlations.

It doesn’t remove the problem, but we start taming it.

My approach:
- is the measurement problem really new?
- aren’t we asking too much to quantum theory?

Deterministic chaos ?



X binary = $0.b_1b_2b_3b_4....b_n.....$

$\rightarrow$ X binary = $0.b_2b_3b_4....b_n.....$

Whether x lies in the left half or the right half after n steps depends on the n^{th} bit, b_n, of the initial condition x.

Is the billionth bits "physically real"? The question is not whether the billionth bit can be measured, but whether it corresponds to something physical?

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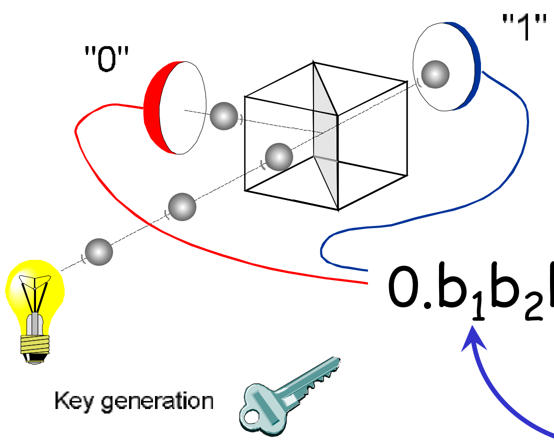
Typical real numbers

- The bi[...] structu[...]n as quanti[...]
- One re[...]o all questi[...]an langua[...]

When we say:
Let $x_0 \in \Re$
what we effectively say is:
let x_0 denote an infinite amount of information.

- The only good way of thinking of a typical real number is the unlimited string of outcomes of a true random number generator.



$0.b_1b_2b_3b_4....b_n.....$

typical real number

G. Chaitin, The Labyrinth of the Continuum, in Meta Math!, Vintage 2008

My intuition of Real Number

0.3906342870591187385492127064927337249582389510
846240185997352602438088375931280495553838500
253876418066556042229443891573679220895949251
6557407035711679933993248067873515575774013122
7431324693922270201711866887060460893183284727
6922985461713919713178282345782969482862331709
948195022204975468668227602507767970267765509
8004162611736118997682498364052959743959340385
72758526775122381194075126706025509511650595705
2699076723890903253959291426547215530138624443
1492940062575082558407172562542821798142848272
4352496992926362434998376619659525125108385861
6044677473288849351659508830828628585264092549
7236099171936642858390197572002307537290953804
45847567821641758542905395011953079755477916723
19340106851299062095688236614693906421190207033
712264516218230979428454131121504352853313616564
87306019819422629712856540790996892145044904136
74815159345054159983821378228976969913337362783

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Classical mathematics Real Number

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0.3906342870591187385492127064927337249582389510846240185997352602438088375931280495553838500253876418066556042229443891573679220895949251

2

7

6922985461713919713178282345782969482862331709

Instead of random events happening as time passes, all the randomness is coded in the initial condition.

Instead of God playing dice as time passes, God played all dies at the big-bang and coded all outcomes in the i.c.

Seems it's the mathematical language that forces us to speak of deterministic chaos !

7236099171936642858390197572002307537290953804
45847567821641758542905395011953079755477916723
19340106851299062095688236614693906421190207033
71226451621823097942845413112150435285331616564
87306019819422629712856540790996892145044904136
74815159345054159983821378228976969913337362783

Whether Newtonian classical mechanics is deterministic or not, is not a scientific question; it depends on the physical significance one associates with mathematical real numbers,
i.e. it depends on the mathematical language: classical mathematics versus intuitionistic mathematics

C. Posy, Mathematical Intuitionism, Cambridge Univ. Press, 2020.
N. Gisin, Indeterminism in Physics and Intuitionistic Mathematics, Synthese 2021, doi.org/10.1007/s11229-021-03378-z

Intuitionistic mathematics

Mathematical real numbers are not Physically real

real numbers are not really real

- **Climate physics uses truncated numbers and stochastic remainders.**

Palmer, T. N. *Nat. Rev. Phys.* **1**, 463–471 (2019).
(see also his Rational Q Mechanics & implications for Q computation)

- Statements about the future can be neither true nor false, but indeterminate. **The law of the excluded middle does not hold**.

The mathematical language we speak has a huge influence on the world-view that physics presents to us.

C. Posy, Mathematical Intuitionism, Cambridge Univ. Press, 2020.
N. Gisin, *Mathematical languages shape our understanding of time in physics*, Nature Physics 16, 114-119 (2020)
N. Gisin, Indeterminism in Physics and Intuitionistic Mathematics, Synthese 2021, doi.org/10.1007/s11229-021-03378-z

The Classical measurement problem

- When do new bits in the series $0.b_1b_2b_3b_4 \ldots b_n \ldots$ gain determined values?
- Precisely, when the classical bits in the given series $0.b_1b_2b_3b_4 \ldots b_n \ldots$ gain relevance.
- More precisely, it is not the value of individual bits that gain determinate values, but information that gets spontaneously created, i.e. new structure of the world becomes real (true becoming).
- This is perfectly in line with the Q info slogan: information is physical.

N. Gisin, *Real Numbers are the Hidden Variables of Classical Mechanics*, Quantum Studies: Mathematics and Foundations 7, 197-201 (2020).

The Classical hidden variables

- Postulating that the initial conditions are faithfully described by classical mathematical real numbers is an elegant way of adding all future results, while making sure that they remain inaccessible for long enough a time.

- $\Rightarrow$ The real numbers are the hidden (truly inaccessible) variables of classical mechanics !

- The hidden variables of classical mechanics are non-contextual, no $\hbar$.

N. Gisin, *Real Numbers are the Hidden Variables of Classical Mechanics*, Quantum Studies: Mathematics and Foundations 7, 197-201 (2020).

Real numbers and Bohmian mechanics

- *The fact is that almost all physicists accept real numbers – without noticing that they are hidden variables – while simultaneously reject Bohmian positions as unnecessary.*

Instead of believing in hidden variables, let's accept indeterminacy.

As time passes, new information comes to the world and reduces indeterminacy. This leads to indeterminism and to becoming.

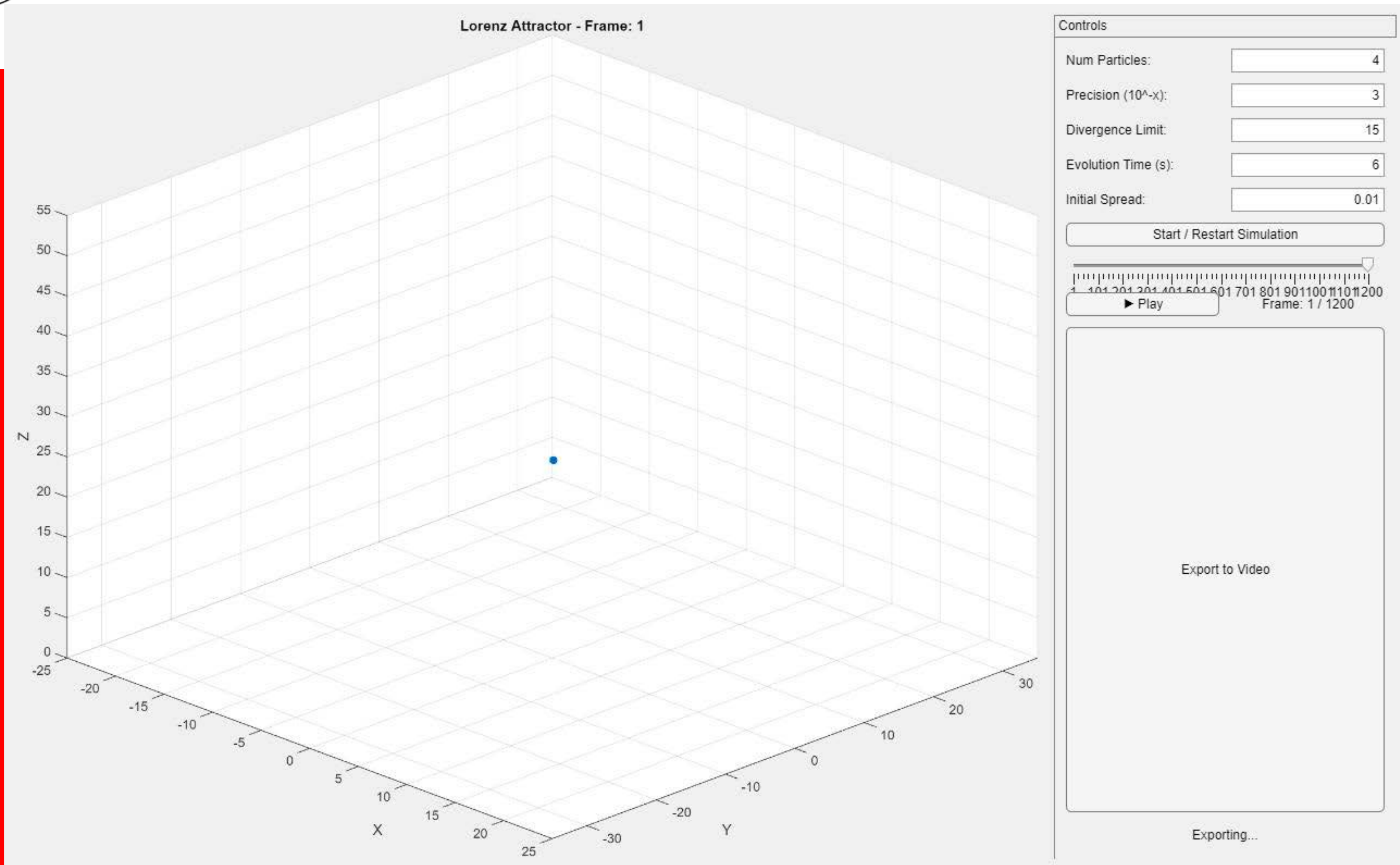
This solution to the classical measurement problem suggests an analogue solution to the deep quantum measurement problem:

Information is physical and this physical information grows with time, though only quantum has incompatible physical quantities, i.e. $\hbar$

N. Gisin, *Real Numbers are the Hidden Variables of Classical Mechanics*, Quantum Studies: Mathematics and Foundations 7, 197-201 (2020). Flavio del Santo & NG, *Which features of quantum physics are not fundamentally quantum but are due to indeterminism?*, Quantum 8, 1267 (2025);
Creative and geometric times, arXiv:2404.06566

Lorenz attractor: a classical beam-spitter

Created by Sajjad Bagdadi



Conclusions

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- All physics theories require an interpretation.
- Physical quantities are never determined with infinite precision: no infinite physical information.
- "Impossible" measurements are localizable, hence possible, but not ideally (not immediately reproducible).
- All joint quantum measurements can be classified from the simplest to the most complex by the number of e-bits necessary to localize them.
- Information is physical and this physical information grows with time.
- This applies also behind the moon: realism ≠ determinism
- Does that solve the deep measurement problem?
- No: a full solution will lead to new physics.
- There is no mechanical solution to the measurement problem